Dissipativity and Performance Analysis of Semiactive Systems with Smart Dampers

Baris ERKUS

supervised by

Prof. Erik A. Johnson

UNIVERSITY OF SOUTHERN CALIFORNIA

NSF CAREER grant CMS 00-94030

METRANS 03-17

Introduction: Structural Control

- Goal
  - solve dynamic problems that cannot be addressed by standard design methodologies such as extreme vibration due to the wind
  - improve seismic performance of vital structures such as hospitals, schools, bridges during an earthquake
- Up to date, various control technologies have been proposed and applied in practice
- Classification: Active, Passive and Semiactive (Housner et al. 1997)
- Our Focus Today: Semiactive Control with *Smart Dampers*

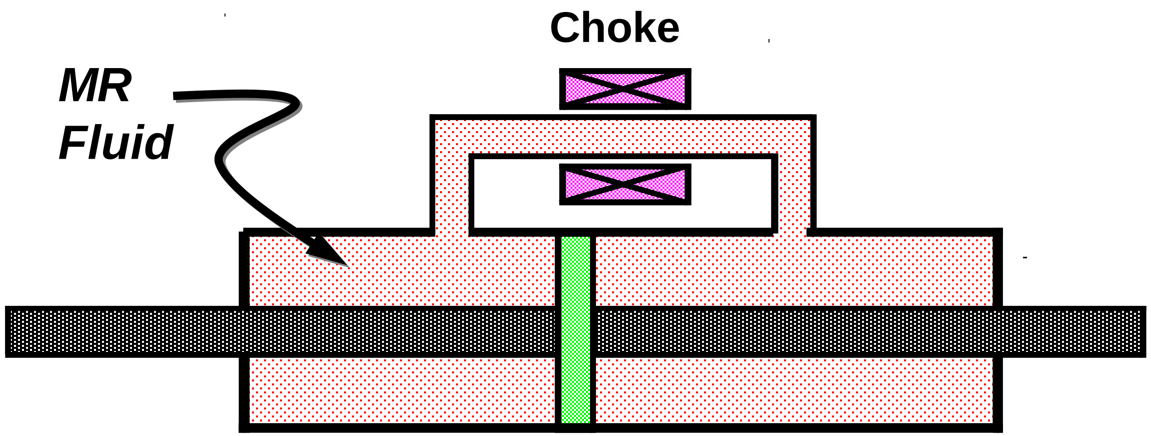
Introduction: Smart Dampers

- Smart dampers are controllable dampers. Example: Magnetorheological (MR) Fluid Damper (Spencer et al. 1996, Yang et al 2002)
- Fundamental property: dissipates energy from the system attached
  - There are no well-established control strategy to account the nonlinearity due to the dissipative nature
- One strategy that is frequently used is clipped optimal control (Dyke et al.)

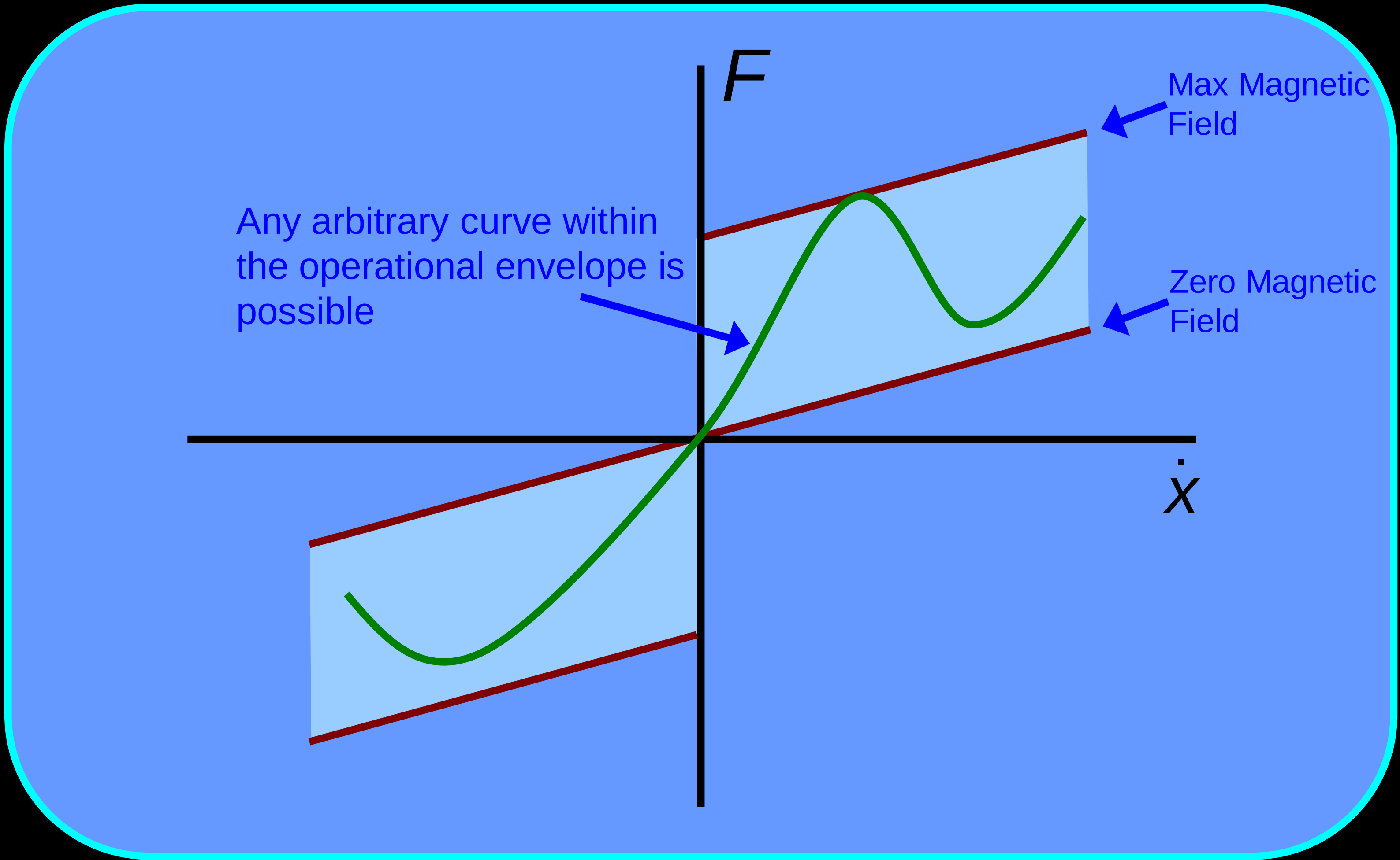
Introduction: Smart Dampers

Introduction: Smart Dampers

Magnetorheological Fluid Damper



Introduction: Smart Dampers



Introduction: Clipped Optimal Control

- A two-stage control strategy

Primary Controller → u_{act} → **Secondary Controller** → i → **Smart Damper** → u

- *System assumed to be fully* ***active and linear***
- E.g. Linear quadratic regulator (LQR)

- *Clipping algorithm*
- *Attempts to make smart damper reproduce u_{act}*

- *Can only exert* ***dissipative*** *forces*

CLIPPED OPTIMAL CONTROL

- How does the fully-active assumption effects the performance of the semiactive system?

→ We must understand the dissipative nature of the *primary controller*

→ *WHAT IS DISSIPATIVITY?*

Introduction: Dissipativity

- Dissipative force: $uv < 0$

  u : force

  v : velocity of the point where the force is applied

- Inaudi (2000) find the probability that a primary control force will be dissipative.
  - Showed by numerical examples that semiactive systems with controllers with high probability of being dissipative can be more effective
  - Christenson (2003, 2004) considered several other examples to justify the results
- Erkus et al. (2002) used percentage of the time that primary control force is dissipative; results are similar
- Johnson (2000) worked on a particular control theory to improve dissipativity, yet the proposed method was not effective

Introduction: Questions

- What is dissipativity? Can we define and quantify dissipativity of a controller and use it to analyze primary controller dissipativity characteristics?
  - *OBJECTIVE* : Introduce dissipativity indices that can quantify dissipative nature of a force
- Can we modify the dissipativity characteristics of a primary controller? Can we improve it?
  - *OBJECTIVE* : Employ dissipativity indices to modify the dissipativity nature of a control force
- Does improving dissipativity of the primary controller improves the final semiactive performance?
  - *OBJECTIVE* : Investigate dissipativity-performance relations for various semiactive systems that are common in civil engineering

Dissipativity Indices

- Strictly dissipative force
  - Consider a control force $u(t)$ applied to a point x_0 on the structure. $u(t)$ is called *strictly dissipative force* if the rate of energy flow is negative for all t ≥ 0.

$$\boxed{\begin{array}{l} u(t)v(t) \le \varepsilon(t) < 0, \quad \text{for all} \quad t \le 0 \Leftrightarrow \\ \qquad\qquad u(t) \ \text{ is strictly dissipative} \end{array}}$$

- Percentage of the dissipative control forces

$$\boxed{D_{\%} = 1 - \frac{1}{N}\sum_{k=0}^{N-1} H[u_{\mathrm{a}}(k\Delta t)v_{\mathrm{d}}(k\Delta t)]}$$

u_{a} : control force

v_{d} : velocity of point x_0

$H[.]$: Heaviside step function

Dissipativity Indices

- Probability that the control force is dissipative (Inaudi, 2000)

$$\boxed{D_{\mathrm{p}} = P[u_{\mathrm{a}} v_{\mathrm{d}} < 0] = \frac{\cos^{-1}(\rho_{u_{\mathrm{a}} v_{\mathrm{d}}})}{\pi}}$$

$\rho_{u_{\mathrm{a}} v_{\mathrm{d}}}$: correlation coefficient between u_{a} and v_{d}

- Assumptions used
  - Excitation is a zero-mean Gaussian white noise

Proposed Dissipativity Indices

- Mean energy flow rate
  - Consider the expected value of the condition for strictly dissipative force:

$$\boxed{D_{\mathrm{e}} = E[u_{\mathrm{a}}(t)v_{\mathrm{d}}(t)]}$$

  - Not a normalized value, may be misleading

- Normalized mean energy flow rate

$$\boxed{D_{\mathrm{ne}} = \frac{E[u_{\mathrm{a}}v_{\mathrm{d}}]}{\sqrt{E[u_{\mathrm{a}}^2]}\sqrt{E[v_{\mathrm{d}}^2]}}}$$

  - Relation between D_{p} and D_{ne} for an LQ problem: D_{ne} is the correlation between u_{a} and v_{d}

Dissipativity Indices: Multiple Controllers

- Following weighted cumulative index is proposed to represent the dissipativity of a system with multiple controllers:

$$\boxed{D^{c} = \sum_{i}^{N} w_i D_i \quad \text{where} \quad w_i = \frac{RMS(u_i)}{\sum_{j}^{N} RMS(u_j)}}$$

Here D_i is the dissipativity of the i th controller

u_i is the i th control force

Dissipativity Indices

- Is there a way to incorporate dissipativity indices into a control theory, such as a linear quadratic regulator problem (LQR), and modify the dissipativity characteristics of the controller.
  - LQR is originally in terms of equality constraints.
  - Represent the LQR in terms of matrix inequalities so that we can incorporate the dissipativity indices into the LQR problem as matrix inequalities.
  - Find the LMI representation of the LQR problem, which allows us to obtain the numerical solution of the LQR problem with a dissipativity index.

LMI Representation of an LQR Problem

- Linear Quadratic Regulator (LQR) Problem
  - Consider a linear time-invariant system and the optimization problem

$$\dot{\mathbf{q}} = \mathbf{Aq} + \mathbf{Bu} + \mathbf{Ew}$$

$$\mathbf{z} = \mathbf{C}_z\mathbf{q} + \mathbf{D}_z\mathbf{u} + \mathbf{F}_z\mathbf{w}$$

Find **K s.t.**

$$\min_{\mathbf{K}} \; E[\mathbf{q}^T\mathbf{Qq} + \mathbf{u}^T\mathbf{Ru} + \mathbf{q}^T\mathbf{Nu} + \mathbf{u}^T\mathbf{N}^T\mathbf{q}]$$

$$\text{subject to} \;\; \dot{\mathbf{q}} = \mathbf{Aq} + \mathbf{Bu} + \mathbf{Ew} \;\; \text{and} \;\; \mathbf{u} = -\mathbf{Kq}$$

$$\mathbf{Q} = \mathbf{C}_z^T\tilde{\mathbf{Q}}\mathbf{C}_z \qquad \mathbf{N} = \mathbf{C}_z^T\tilde{\mathbf{Q}}\mathbf{D}_z + \mathbf{C}_z^T\tilde{\mathbf{N}}$$

$$\mathbf{R} = \tilde{\mathbf{R}} + \mathbf{D}_z^T\tilde{\mathbf{Q}}\mathbf{D}_z + \mathbf{D}_z^T\tilde{\mathbf{N}} + \tilde{\mathbf{N}}^T\mathbf{D}_z$$

$$\tilde{\mathbf{Q}} \geq 0 \qquad \tilde{\mathbf{R}} \geq 0$$

are symmetric weighting matrices

To be well-posed, LQR weights must satisfy

$$\mathbf{W} = \begin{bmatrix} \mathbf{Q} & \mathbf{N} \\ \mathbf{N}^T & \mathbf{R} \end{bmatrix} \geq 0 \quad \text{and} \quad \mathbf{R} > 0$$

LMI Representation of an LQR Problem

- Final Form of the LMI-LQR problem

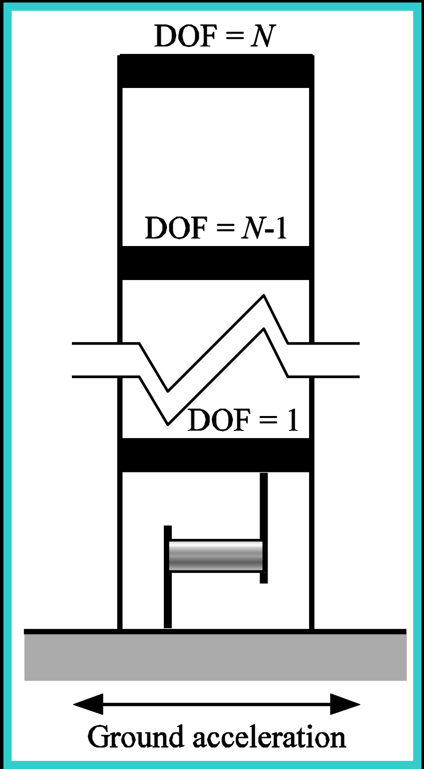
$$\min_{(\mathbf{Y},\mathbf{S},\mathbf{X})} \quad Tr(\mathbf{Q}^{1/2}\mathbf{S}\mathbf{Q}^{1/2}) + Tr(\mathbf{X}) - Tr(\mathbf{Y}\mathbf{N}) - Tr(\mathbf{N}^{T}\mathbf{Y}^{T})$$

$$\text{subject to} \quad \mathbf{AS} - \mathbf{BY} + \mathbf{SA}^{T} - \mathbf{Y}^{T}\mathbf{B}^{T} + \mathbf{EE}^{T} < 0, \quad \begin{bmatrix} \mathbf{X} & \mathbf{R}^{1/2}\mathbf{Y} \\ \mathbf{Y}^{T}\mathbf{R}^{1/2} & \mathbf{S} \end{bmatrix} > 0$$

$$\text{and} \qquad \mathbf{S} = \mathbf{S}^{T} > 0$$

- Let the solution of the LMI-LQR problem be Y_0, S_0 and X_0
- Then, the feedback gain, $F_0 = Y_0(S_0)^{-1}$
- S is called a *Lyapunov Matrix.* In the above problem, $S_0 = P$, where P is the state covariance matrix

Is the LMI-LQR Equivalent to the LQR?



An N-DOF structure

Story mass = 100 tons

Story period, T = 0.5 sec

Modal damping, ζ = 2%

$$\mathbf{Q} = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix}$$

$$\mathbf{R} = \eta, \quad \mathbf{N} = \mathbf{0}$$

Compare the control gains and covariance matrices of LMI-LQR and LQR with the following error indices:

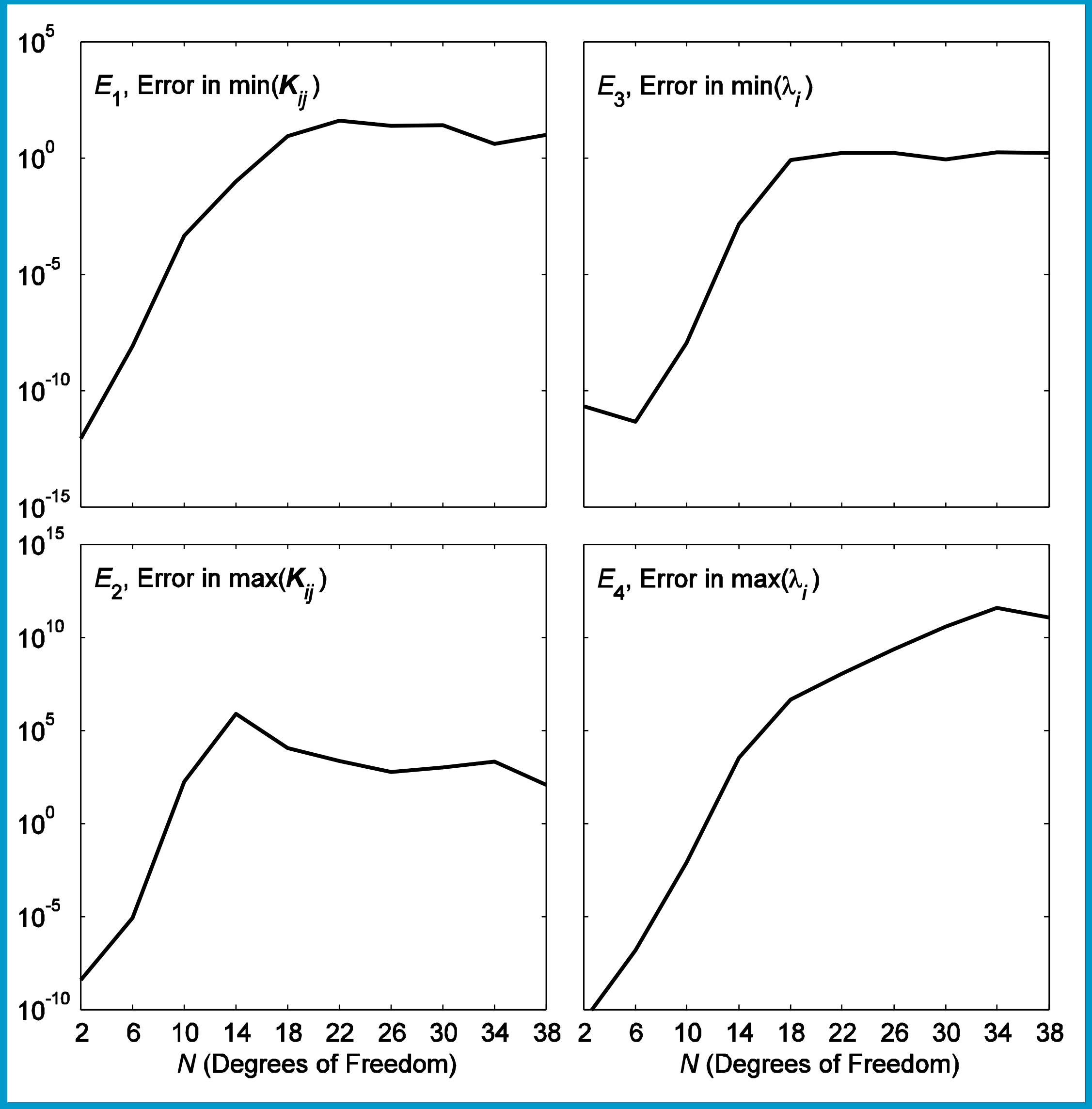
$$E_1 = \frac{\min(|K_{ij}^{\mathrm{LMI}}|) - \min(|K_{ij}^{\mathrm{LQR}}|)}{\min(|K_{ij}^{\mathrm{LQR}}|)},$$ min element of control gain

$$E_2 = \frac{\max(|K_{ij}^{\mathrm{LMI}}|) - \max(|K_{ij}^{\mathrm{LQR}}|)}{\max(|K_{ij}^{\mathrm{LQR}}|)},$$ max element of control gain

$$E_3 = \frac{\min(|\lambda_{ij}^{\mathrm{LMI}}|) - \min(|\lambda_{ij}^{\mathrm{LQR}}|)}{\min(|\lambda_{ij}^{\mathrm{LQR}}|)},$$ min eigenvalue of covariance

$$E_4 = \frac{\max(|\lambda_{ij}^{\mathrm{LMI}}|) - \max(|\lambda_{ij}^{\mathrm{LQR}}|)}{\max(|\lambda_{ij}^{\mathrm{LQR}}|)},$$ max eigenvalue of covariance

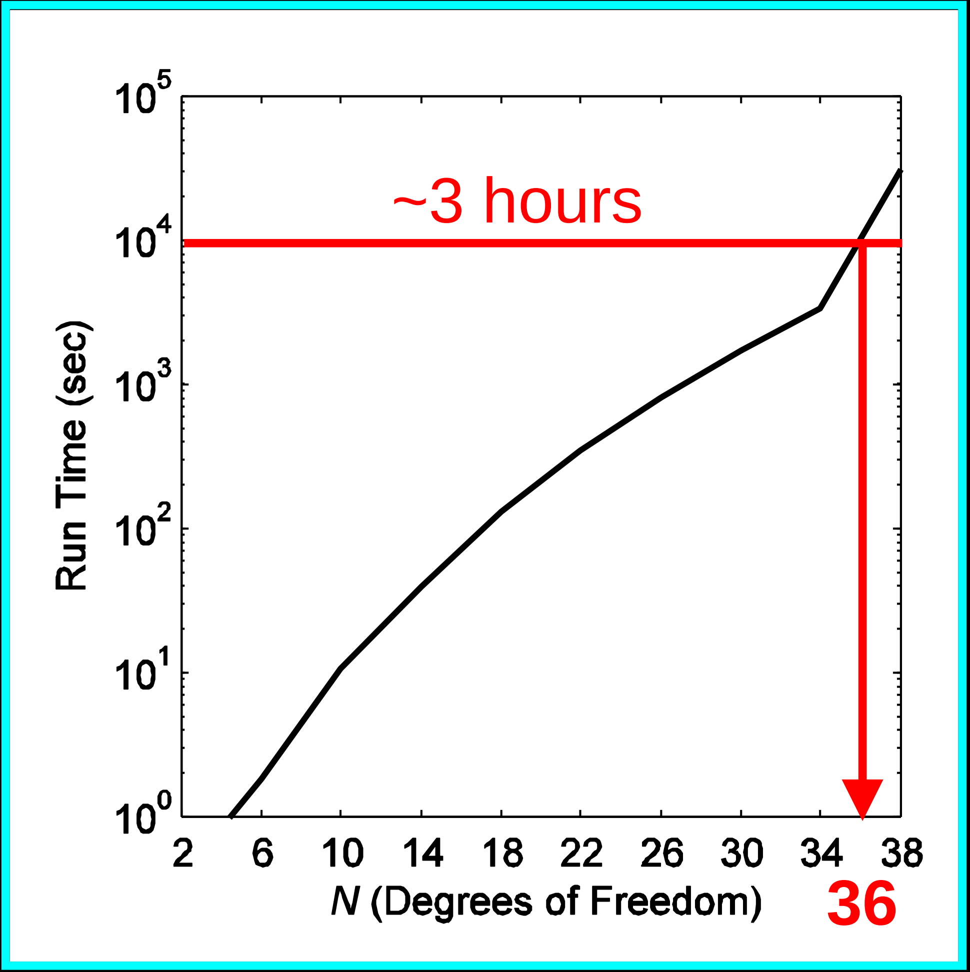
Is the LMI-LQR Equivalent to the LQR?



- MATLAB LMI Control Toolbox
- As the system DOF increases, LMI-solver becomes inefficient
- A maximum of 10 DOF is suggested to be used

Is the LMI-LQR Equivalent to the LQR?

- Time efficiency of the LMI solver



- As the system DOF increases, LMI-solver becomes very time-inefficient

Dissipativity Constraints

- D_e-based constraint:

$$\boxed{-\mathbf{F}\mathbf{S}\mathbf{C}_v^T < \gamma_e^L}$$

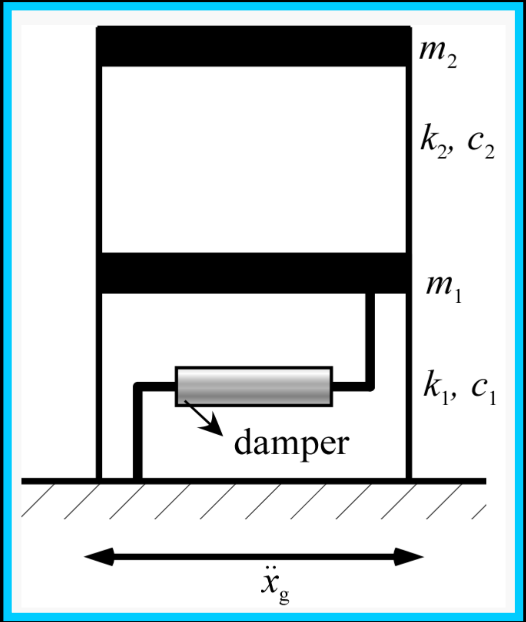
LMI-Toolbox can be used

- D_{ne}-based constraint:

$$\boxed{\frac{-\mathbf{F}\mathbf{S}\mathbf{C}_v^T}{\sqrt{\mathbf{F}\mathbf{S}\mathbf{F}^T}\sqrt{\mathbf{C}_v\mathbf{S}\mathbf{C}_v^T}} < \gamma_{ne} \quad \text{where} \quad -1 \leq \gamma_{ne} \leq 1}$$

LMI-Toolbox cannot be used; an iterative method is proposed

2-DOF Systems: Shear Building (D_e)



$m_1 = m_2 = 100$ tons

Story period, $T = 0.5$ sec

Modal damping, $\zeta = 2\%$

Ideal Damper

$$u_d = \begin{cases} u_a, & u_a v_d < 0 \\ 0, & u_a v_d \geq 0 \end{cases}$$

Outputs to be minimized

→Absolute floor accelerations

→Storey drifts

$$J = J_d + \frac{\beta}{\alpha} J_a + \frac{\eta}{\alpha} \sigma_u^2 \quad \text{(DRIFT + ABS. ACCEL)}$$

Performance Ind.:

$$J_d = \sigma_{x_1}^2 + \sigma_{x_2 - x_1}^2 \quad \text{(DRIFT)}$$

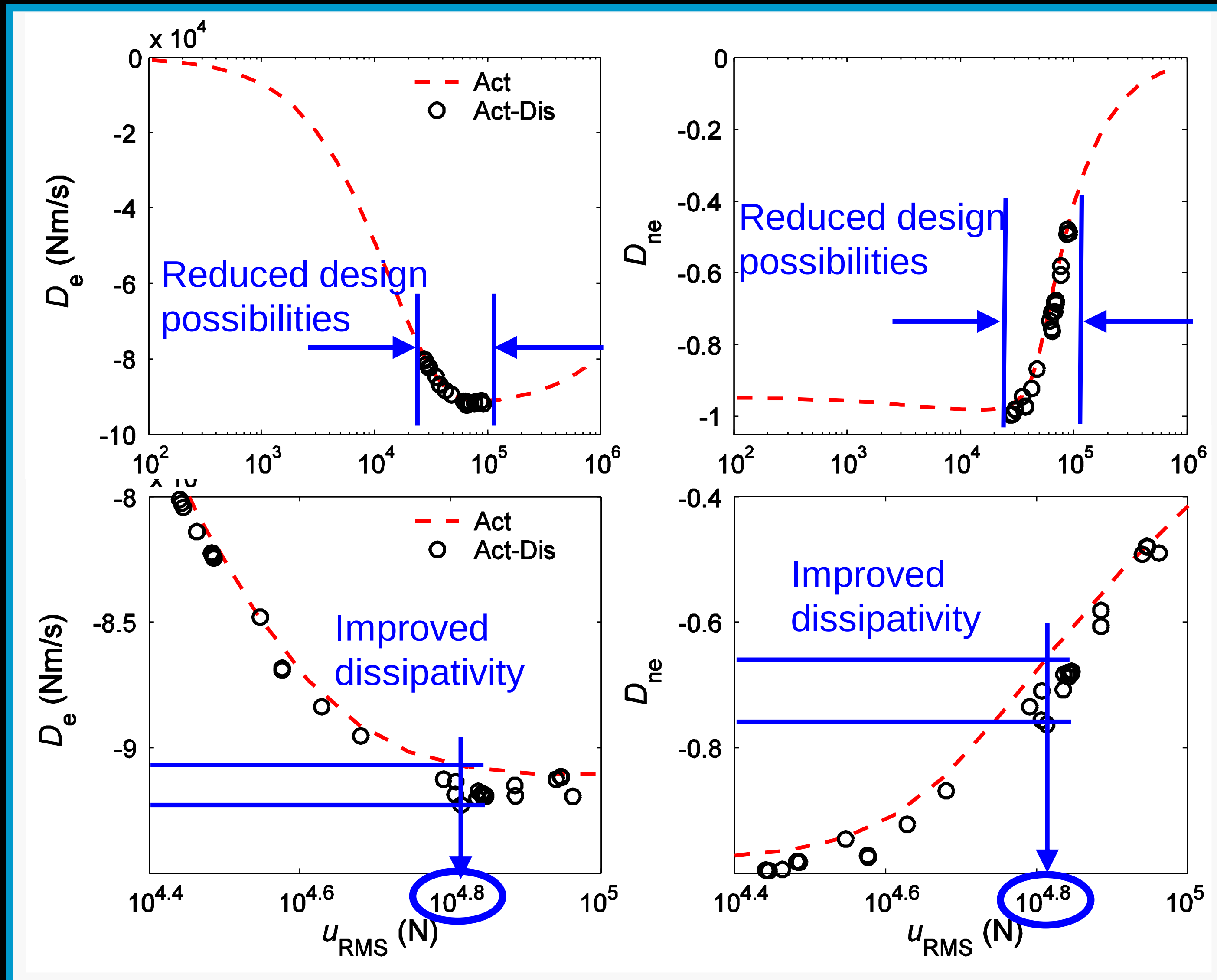
$$J_a = \frac{1}{\omega_n^4} \left(\sigma_{\ddot{x}_1^{abs}}^2 + \sigma_{\ddot{x}_2^{abs}}^2 \right) \quad \text{(ABS. ACCEL)}$$

2-DOF Systems: Shear Building (D_e)

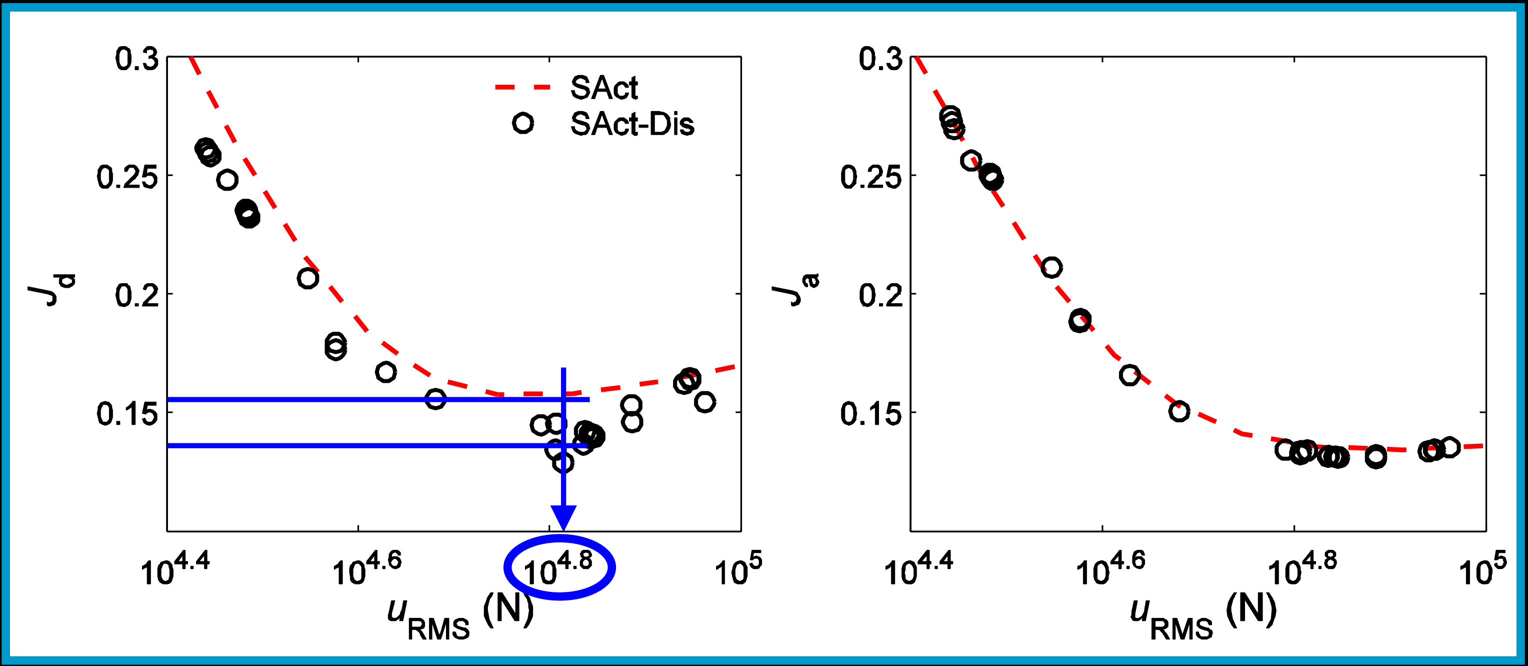
$\alpha : 10^{-5} \rightarrow 10^{5}$

$\beta : 1000$

$\eta : 10^{-12}$



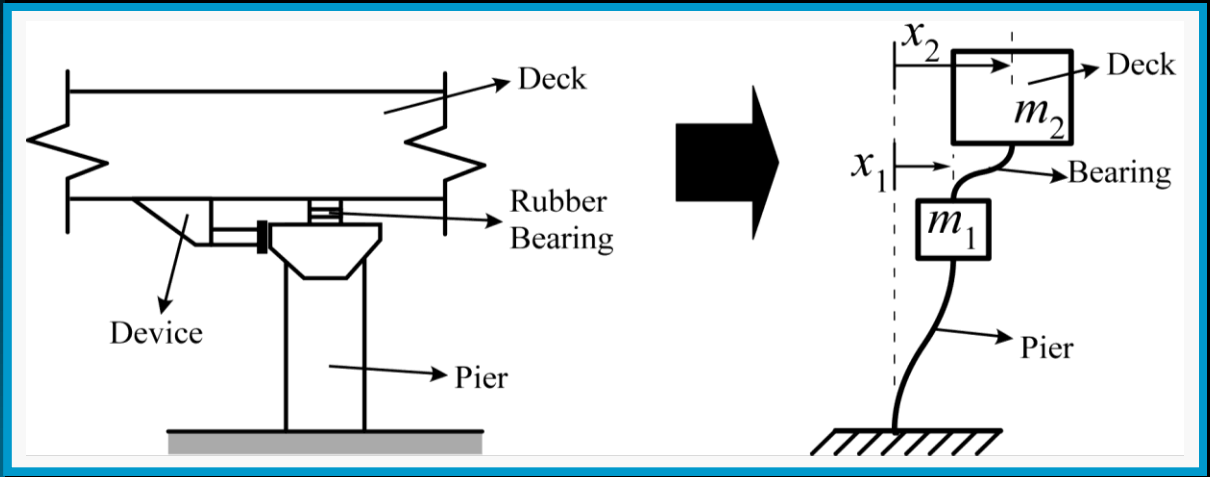
2-DOF Systems: Shear Building (D_e)



Systems	$D_{\%}$(%)	J_d	J_a
SAct	0.760	0.160	0.135
SAct-Dis	0.793	0.129	0.134

20% Improvement

2-DOF Systems: Highway Bridge (D_e)



Mass ratio, $m_2/m_1 = 5$

Pier natural period, T = 0.5 sec

Pier damping, ζ = 5%

Bearing damping

Active Systems → Zero

Uncontr. → 196 kNsec/m,

A realistic MR damper model is used.
Control design is selected such that it produces low dissipative control forces

→ Numerical simulations shows that realistic MR damper reduces dissipativity further, reducing the overall semiactive performance

2-DOF Systems: Shear Building (D_{ne})

Iterative solution

- Start with an initial $\mathbf{S}^i$ and $\mathbf{F}^i$ and solve the following problem
- Iterate over $\mathbf{F}$ and $\mathbf{S}$

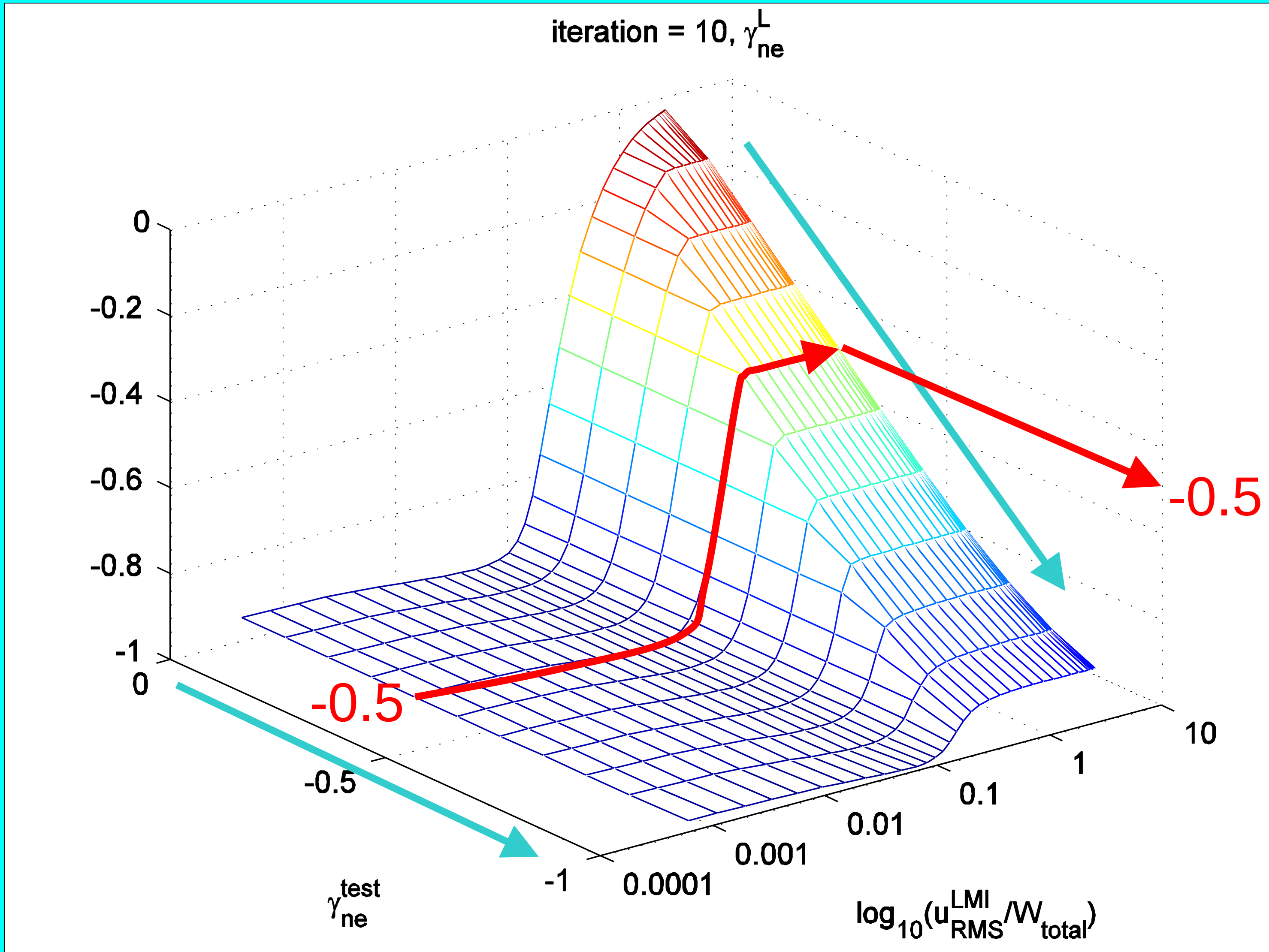
$$\min_{(\mathbf{Y},\mathbf{S},\mathbf{X})} \; Tr(\mathbf{Q}^{1/2}\mathbf{S}\mathbf{Q}^{1/2}) + Tr(\mathbf{X}) - Tr(\mathbf{YN}) - Tr(\mathbf{N}^{\mathsf{T}}\mathbf{Y}^{\mathsf{T}})$$

$$\text{subject to} \quad \mathbf{AS} - \mathbf{BY} + \mathbf{SA}^{\mathsf{T}} - \mathbf{Y}^{\mathsf{T}}\mathbf{B}^{\mathsf{T}} + \mathbf{EE}^{T} < 0, \quad \begin{bmatrix} \mathbf{X} & \mathbf{R}^{1/2}\mathbf{Y} \\ \mathbf{Y}^{\mathsf{T}}\mathbf{R}^{1/2} & \mathbf{S} \end{bmatrix} > 0$$

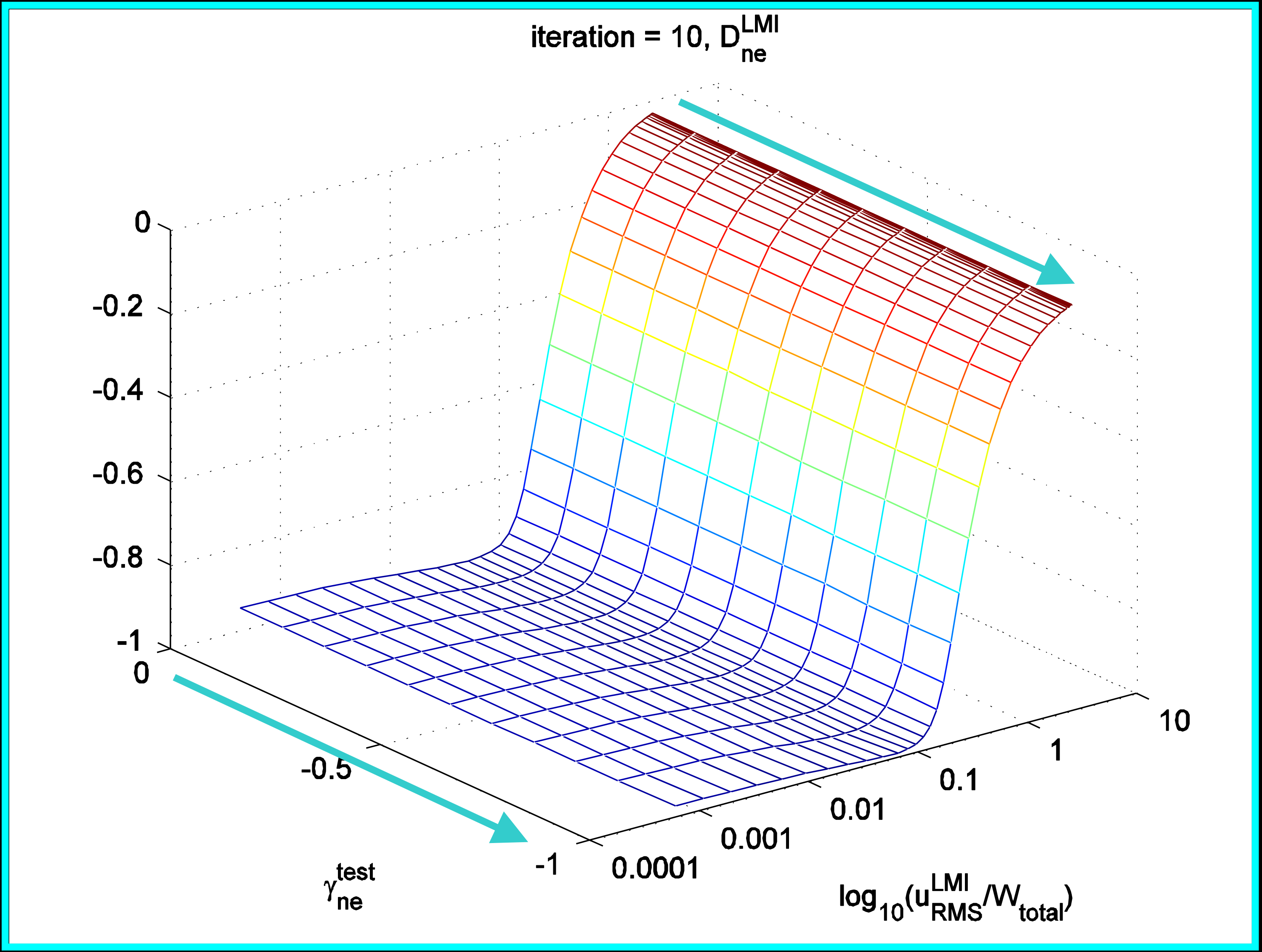
$$\frac{-\mathbf{FSC}_v^{\mathsf{T}}}{\sqrt{\mathbf{F}^i\mathbf{S}^i(\mathbf{F}^i)^{\mathsf{T}}}\sqrt{\mathbf{C}_v\mathbf{S}^i\mathbf{C}_v^{\mathsf{T}}}} < \gamma_{ne}^{test}$$

For known $\mathbf{F}^i$ $\mathbf{S}^i$, this is an LMI

2-DOF Systems: Shear Building (D_{ne})

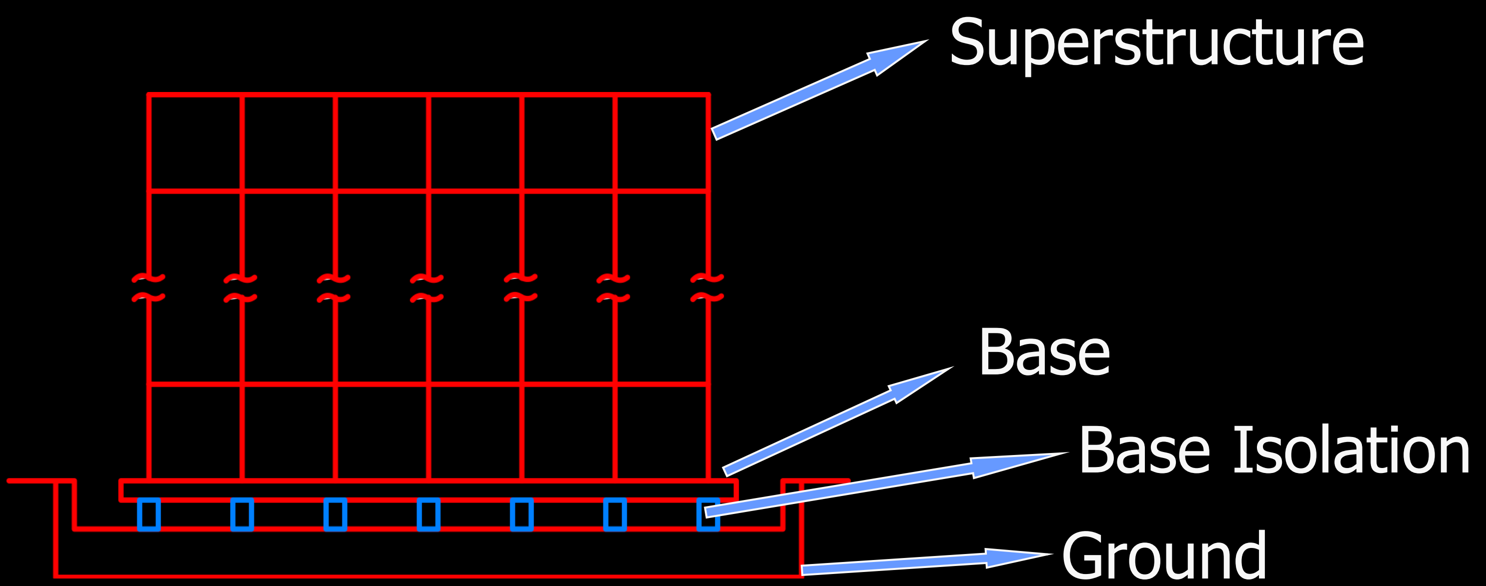


2-DOF Systems: Shear Building (D_{ne})



Base Isolated Benchmark Structure

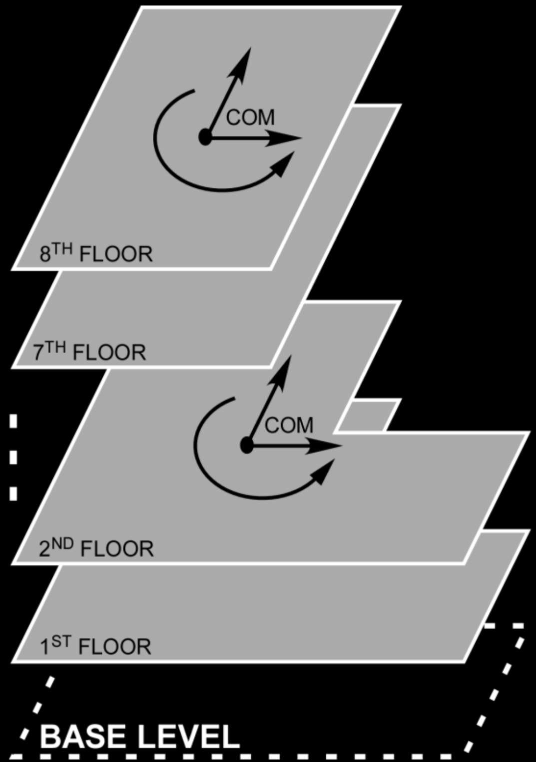
- 8-story steel braced frame building
- Superstructure rests on a rigid concrete base
  - Columns, beams, bracings and rigid slab
  - First to sixth floors are L-shaped, seventh and eighth floors are rectangular
- Rigid base is isolated from the ground by the isolation layer
  - Rubber bearings, lead-rubber bearings, sliding friction bearings



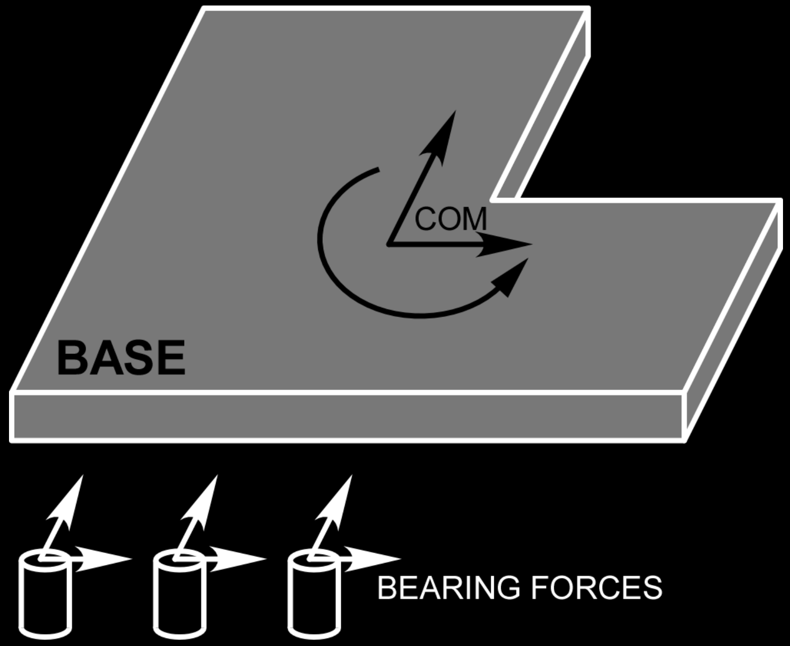
Representative figure of benchmark structure

BM Structure: Modelling

- Superstructure
  - Three dimensional: x, y and rotational DOF at each floor



- Base and the isolation layer
  - 3 DOFs at the center of mass of the base
  - Various linear and nonlinear isolator elements



BM Structure: LQ-based Design

- Outputs to be minimized are
  - Absolute floor accelerations
  - Corner isolator drifts

- The LQ weight matrices:

$$\tilde{\mathbf{Q}} = \begin{bmatrix} a\alpha & \mathbf{0} \\ \mathbf{0} & b\beta \end{bmatrix}, \quad \tilde{\mathbf{R}} = r\,\mathbf{I}, \quad \tilde{\mathbf{N}} = \mathbf{0}$$

a and *b* determines the relative importance of abs. floor acceleration and corner isolator drifts

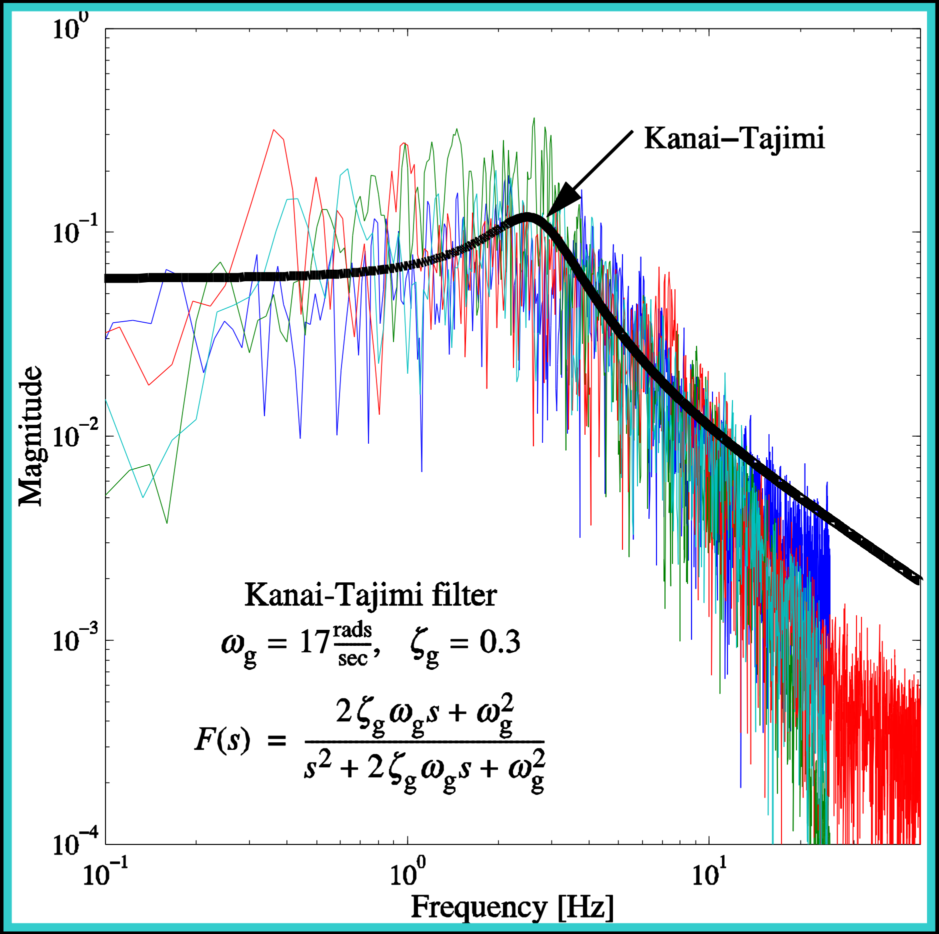
- Kalman filter is used to estimate the states

- Kanai-Tajimi filter is augmented to the system and the augmented system is used in the LQ design

$$G_{\mathrm{KT}}(s) = \frac{2\zeta_{\mathrm{g}}\omega_{\mathrm{g}}s + \omega_{\mathrm{g}}^2}{s^2 + 2\zeta_{\mathrm{g}}\omega_{\mathrm{g}}s + \omega_{\mathrm{g}}^2} \qquad \zeta_{\mathrm{g}} = 0.3, \quad \omega_{\mathrm{g}} = 17 \text{ rad/sec}$$

BM Structure: Kanai-Tajimi Filter

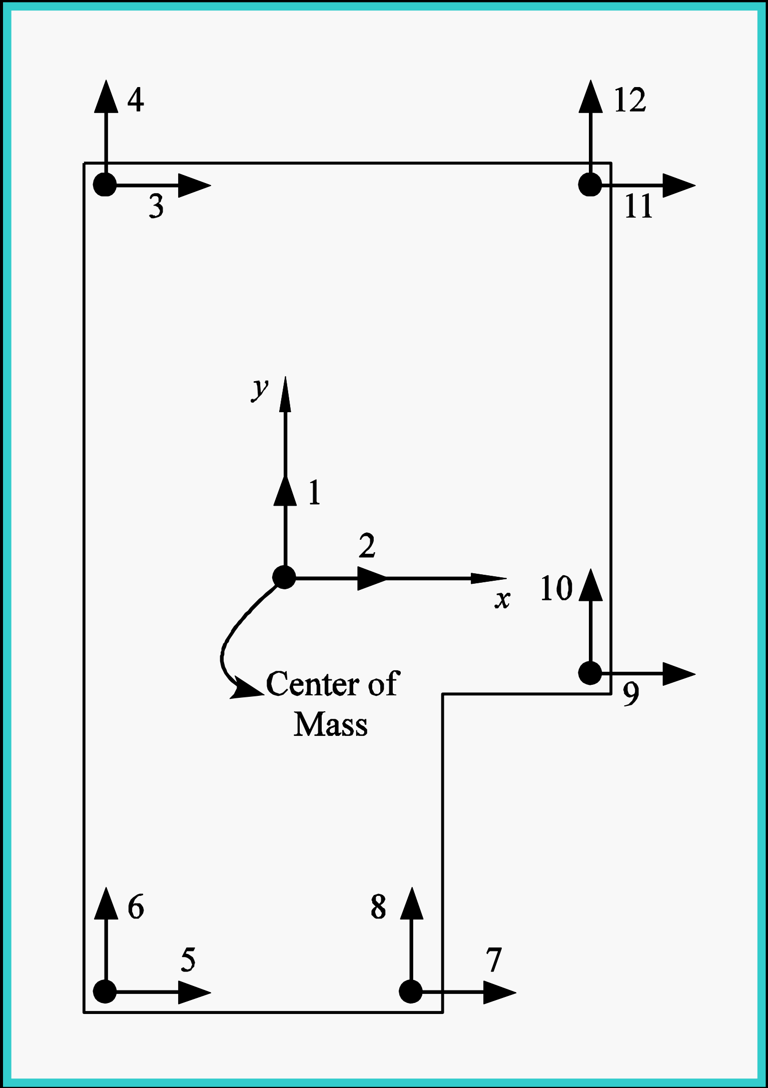
- Filter parameters



(Figure is taken from Ramallo et al. (2002))

- Filter parameters are given by Ramallo et al. (2002)
- Four representative earthquakes (El Centro, Hachinohe, Kobe and Northridge) are used

BM Structure: Controller Locations



- 12 controllers are used
- In an active system, the controllers are assumed to be fully active
- In a semiactive system 20-ton MR dampers are used. Damper forces are magnified by a factor to have comparable force levels to the primary control force

BM Structure: Performance Indices

J	DEFINITION
J_1	Peak Base Shear
J_2	Peak First Floor Shear
J_3	Peak Base Drift
J_4	Peak Interstory Drift
J_5	Peak Absolute Floor Acceleration
J_6	Peak Controller Force
J_7	RMS Base Drift
J_8	RMS Absolute Floor Acceleration
J_9	Energy Absorbed by the Control Devices
J_{10}	Peak Controller Force Normalized by Structure Weight
J_{11}	RMS Floor Drifts

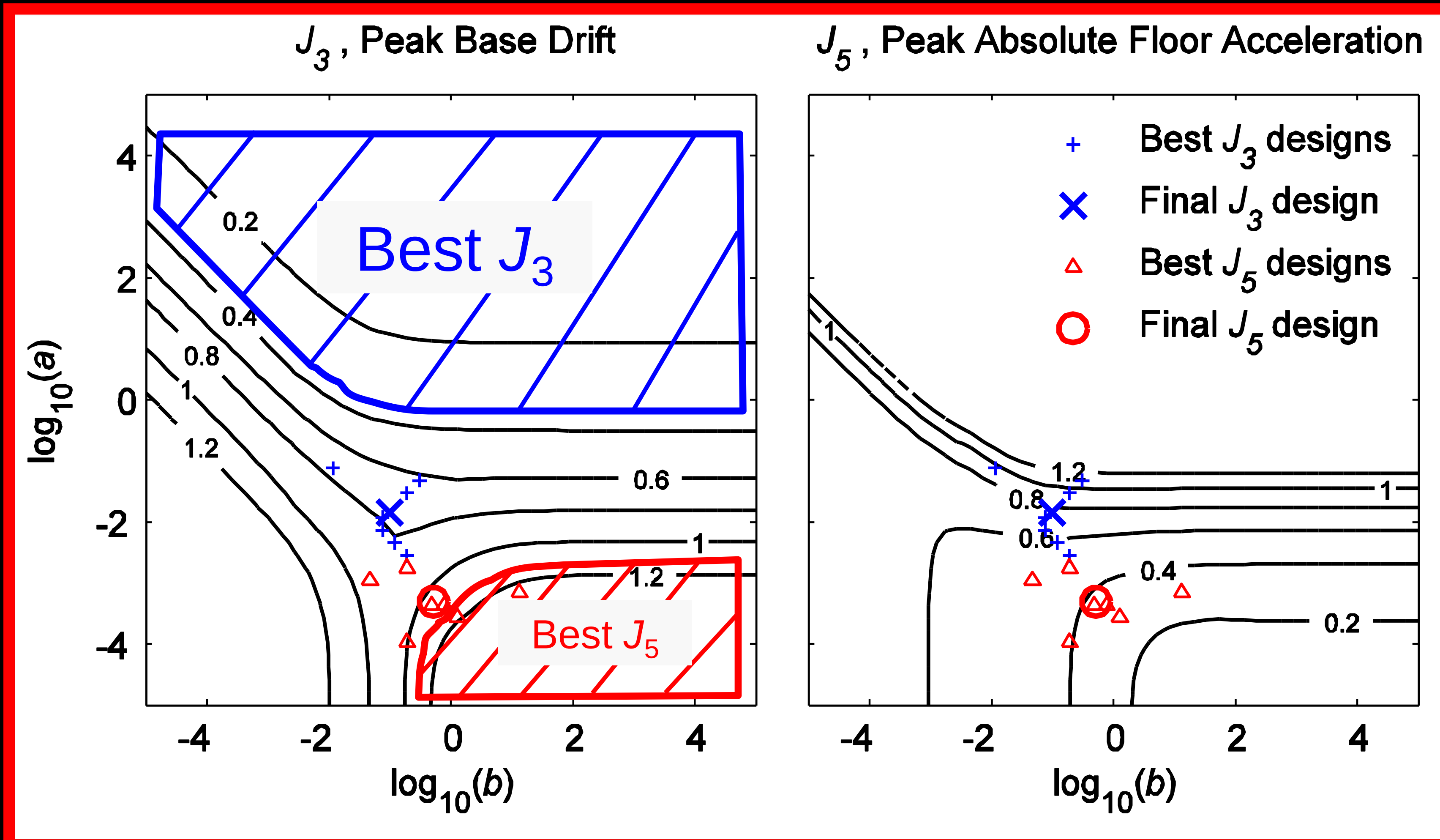
BM Structure: Control Designs

- Two control designs are considered
  - DES1: Best *Peak Base Drift* (J_3)
  - DES2: Best *Peak Absolute Floor Acceleration* (J_5)
  - In the parametric strudies, following conditions will be used

	DES1	DES2
J_3	min (J_3)	$J_3 < 1.0$
J_4	$J_4 < 1.0$	
J_5	$J_5 < 1.0$	min (J_5)
J_{10}	$J_{10} < 0.15$	

Linear BM structure: Active System

- Newhall Earthquake



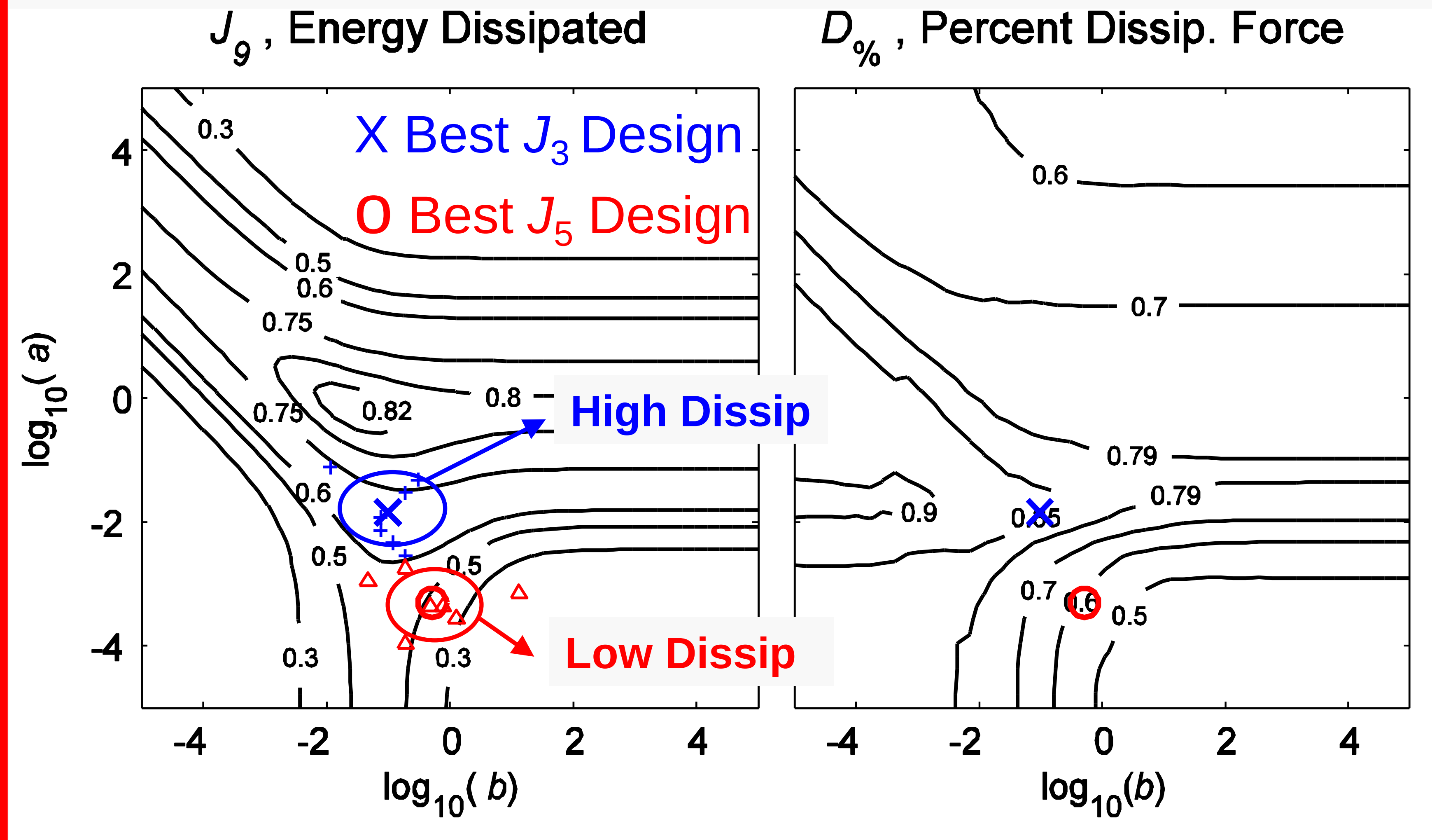
Linear BM Structure: Performance Ind.

- Newhall Earthquake

		BEST J_3 (DES1)		BEST J_5 (DES2)	
J	**Definition**	**ACT**	**SACT**	**ACT**	**SACT**
J_1	Peak Base Shear	0.969	0.979	0.581	1.058
J_2	Peak First Floor Shear	1.071	1.188	0.611	1.240
J_3	**Peak Base Drift**	**0.764**	**0.854**	1.047	0.972
J_4	Peak Interstory Drift	0.689	1.103	0.379	1.123
J_5	**Peak Absolute Floor Acceleration**	0.750	1.087	**0.372**	**1.157**
J_6	Peak Controller Force	0.624	0.552	0.488	0.473
J_7	RMS Base Drift	0.774	0.848	1.034	1.025
J_8	RMS Absolute Floor Acceleration	0.804	1.110	0.423	1.066
J_9	Energy Absorbed by the Con. Dev.	**0.701**	**0.718**	**0.477**	**0.651**
J_{10}	Normalized Peak Controller Force	0.111	0.099	0.087	0.084
J_{11}	RMS Floor Drifts	0.876	1.026	0.502	1.003

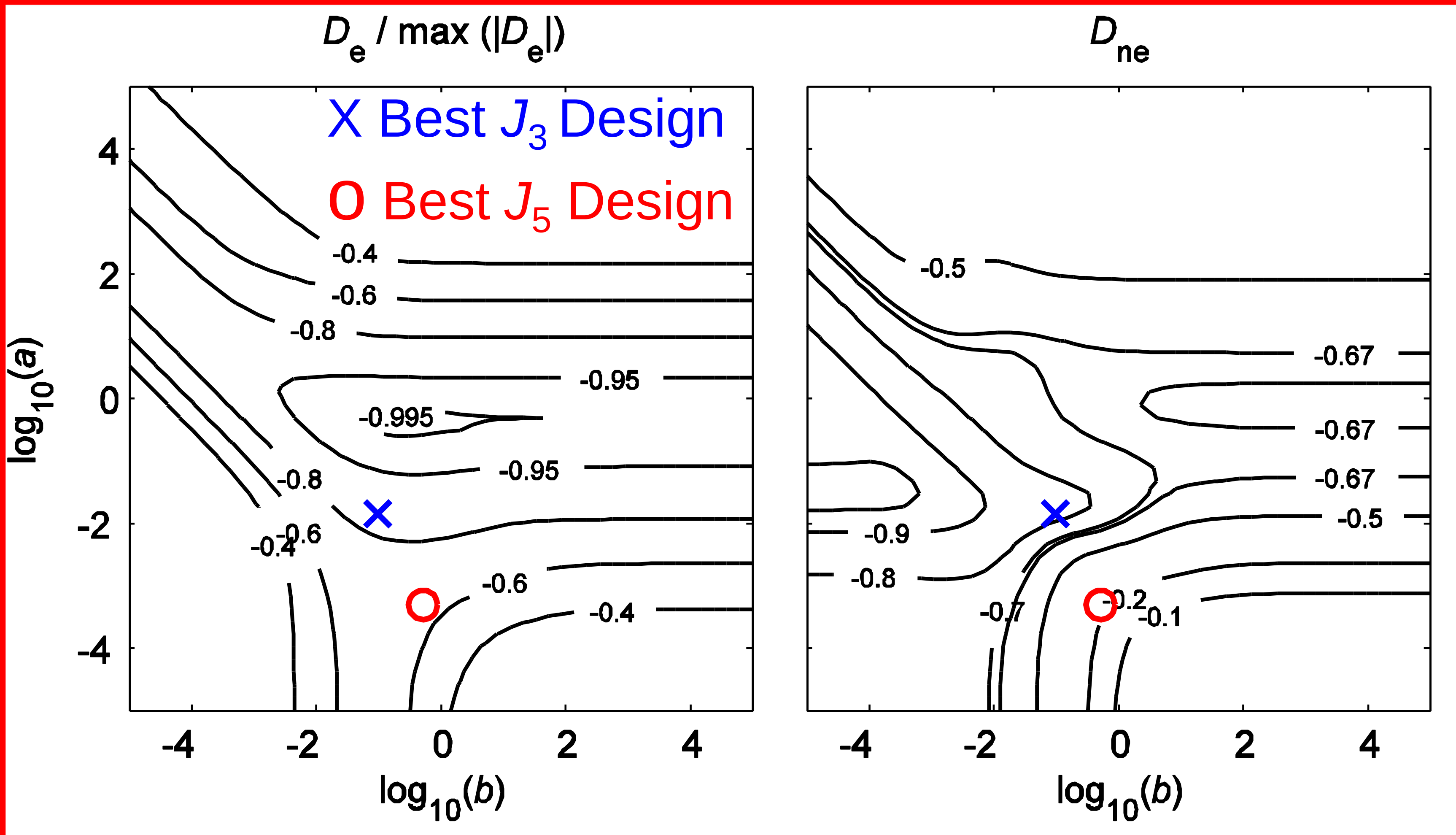
Linear BM Structure: Active System

- Newhall Earthquake



Linear BM Structure: Active System

- Stochastic dissipativity Indices

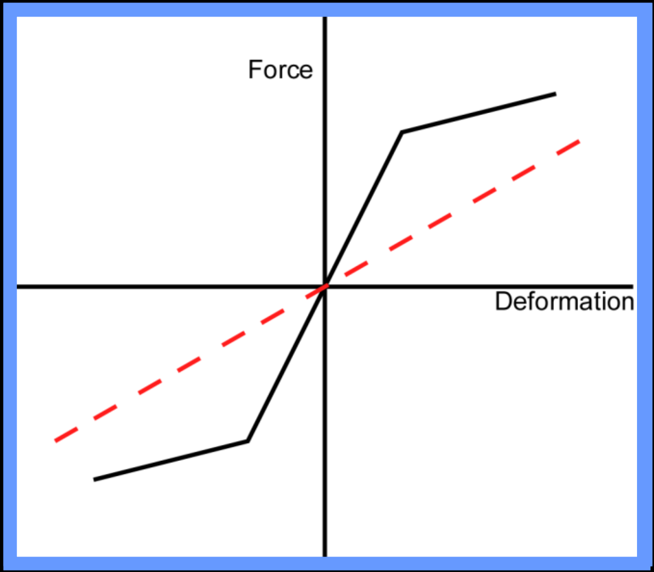


Nonlinear BM Structure

- No efficient control theory is available for nonlinear structures that are common in civil engineering
- Equivalent linearization methods is utilized to obtain and equivalent liner model (ELM) of the nonlinear structure
- An iterative method is proposed to obtain an ELM
  - Nonlinear BM structure has bilinear isolators
  - ELM will have linear isolators

Nonlinear BM Structure: Control Design

1. START : Pick a linear model
   – Select linear stiffness for nonlinear bearings: preyield stiffness
2. Design a linear controller
3. Numerical analysis of both controlled nonlinear and linear structure models for a specific ground motion data
4. Iteration decision
   – Compute RMS force of nonlinear bearings
   – Compute γ_f
   – Update stiffness
   – No change in the damping
5. Go to step 2 END : an error convergence criteria is satisfied.



$$\gamma_{\mathrm{f}} = \frac{\mathrm{RMS}[F_{\mathrm{b}}^{\mathrm{nonlin}}]}{\mathrm{RMS}[F_{\mathrm{b}}^{\mathrm{lin}}]}$$

$$K_{i+1}^{\mathrm{linear}} = \gamma_{\mathrm{f}}^{i} K_{i}^{\mathrm{linear}}$$

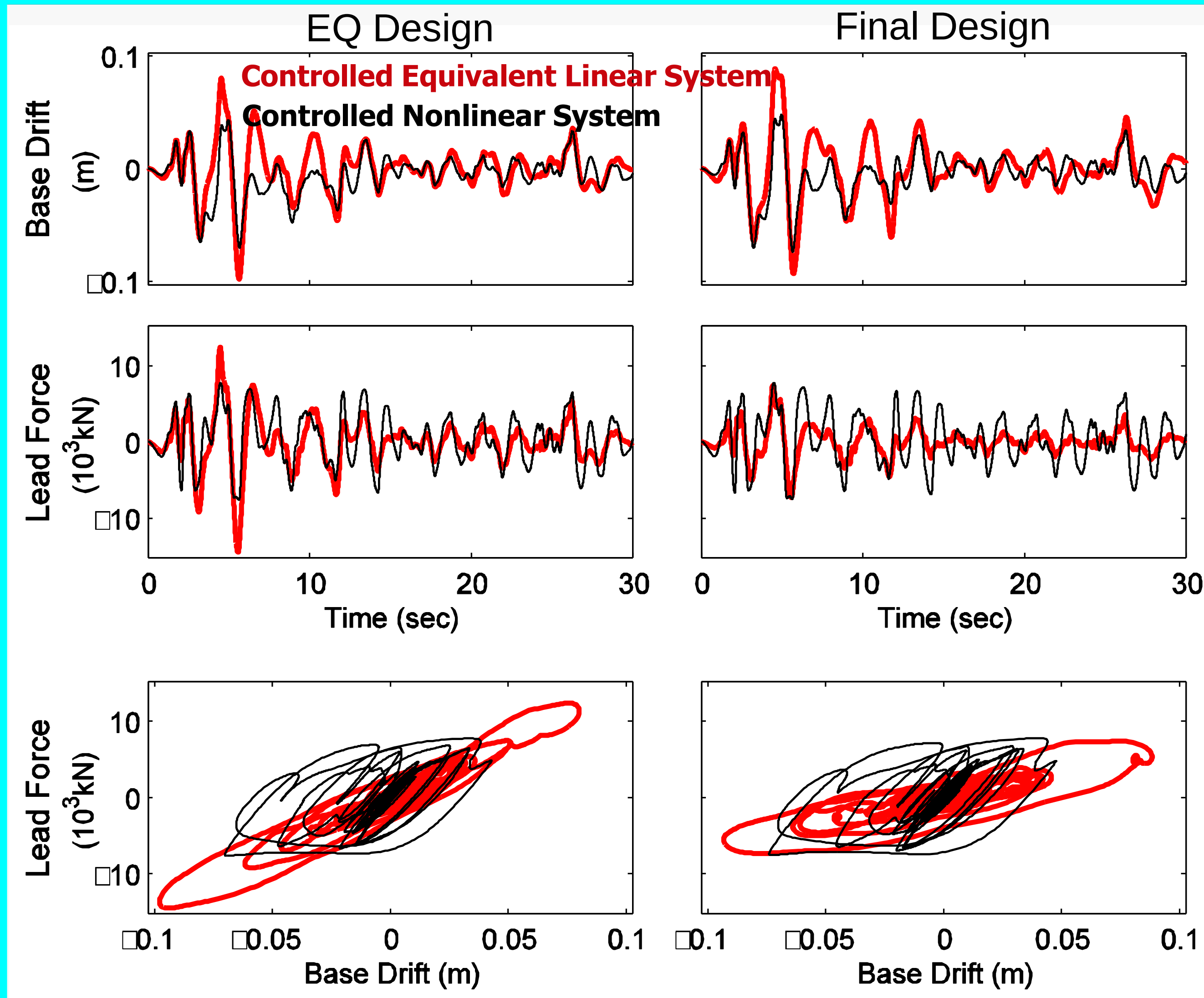
Nonlinear BM Structure: ELM vs Nonlinear

- El Centro Earthquake

Base Disp.

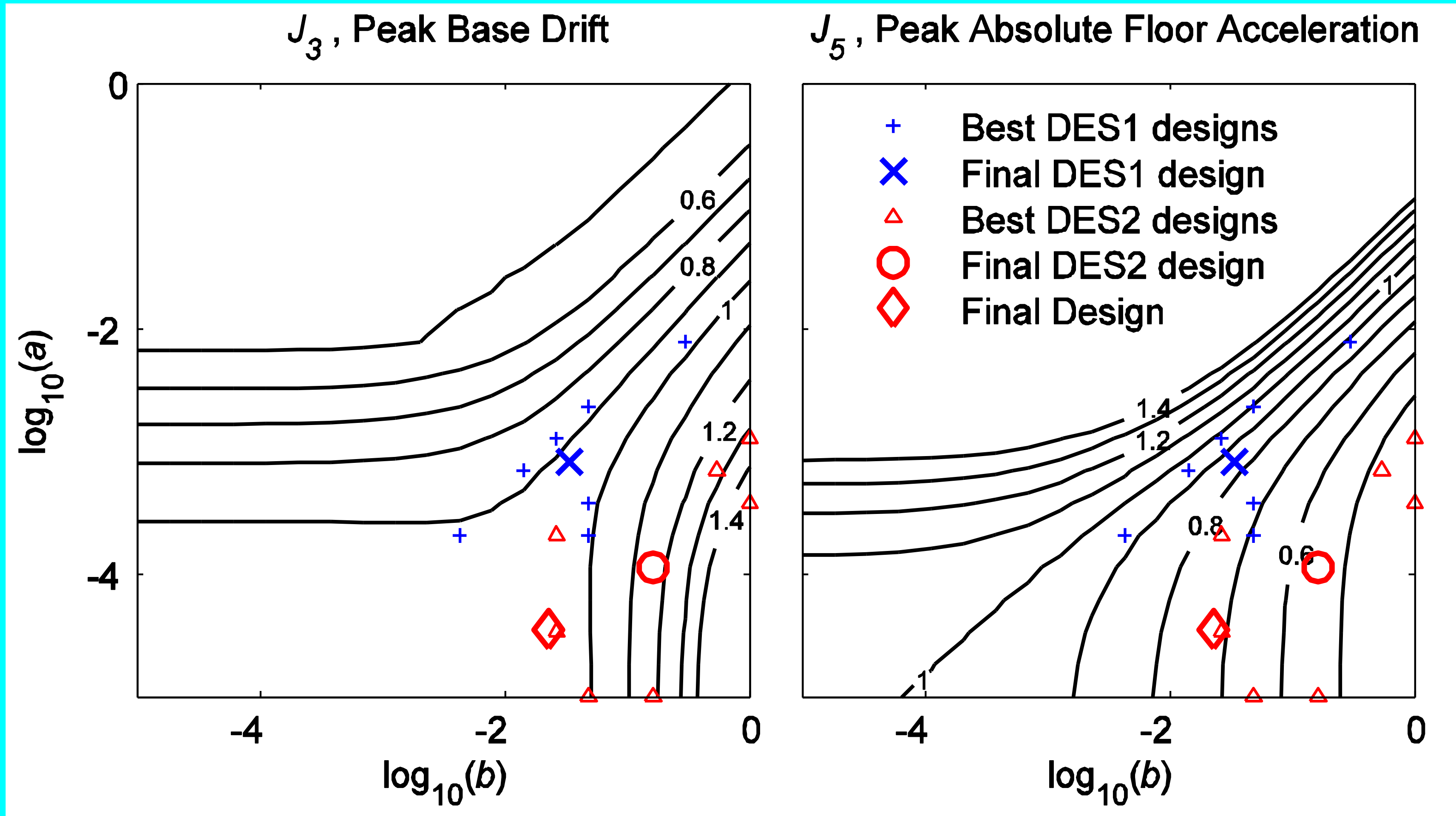
Nonlin Force

Hysteretic Behavior of the Lead Plugs Only



Nonlinear BM structure: Active System

- Newhall Earthquake



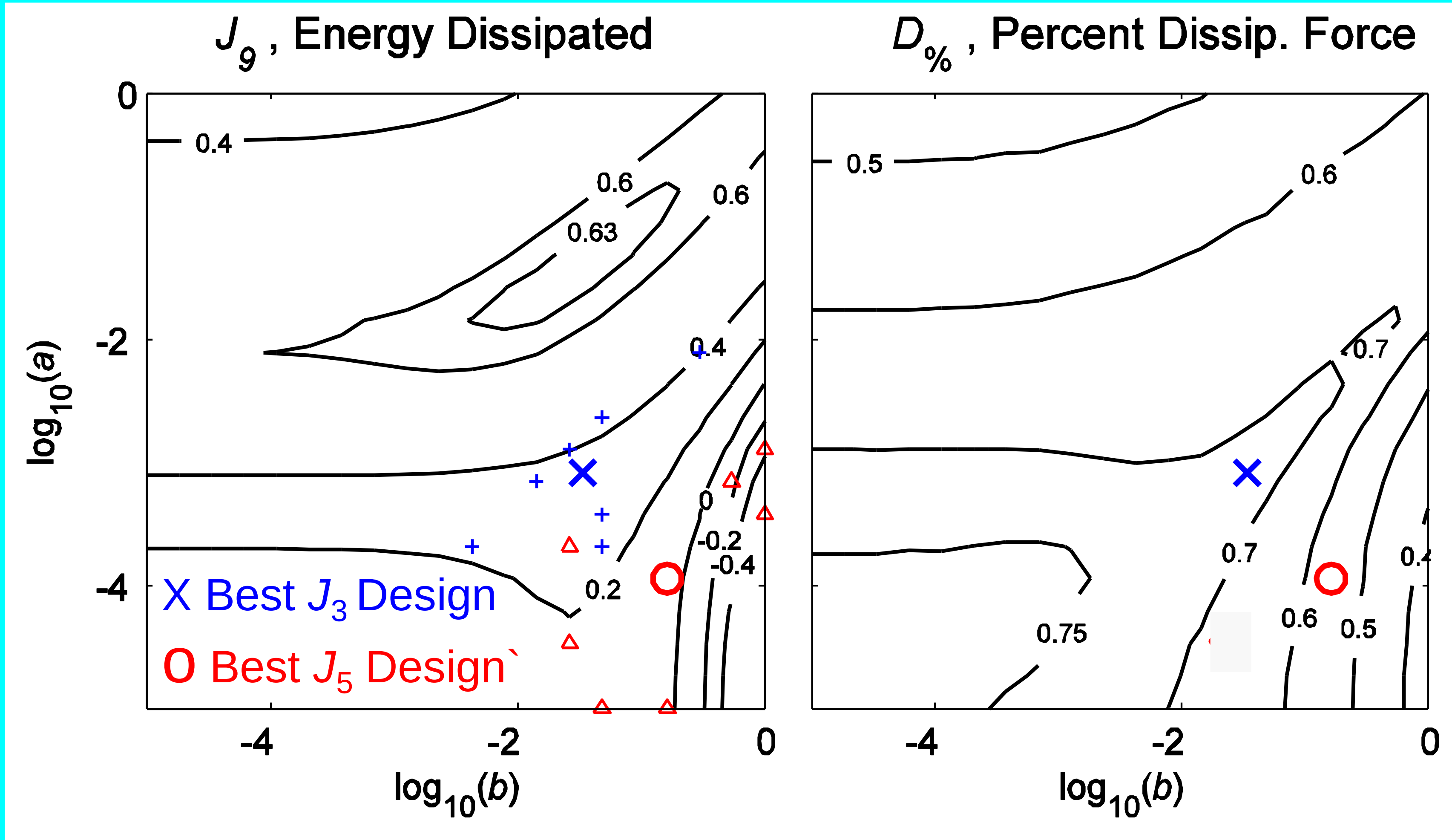
Linear BM Structure: Performance Ind.

- Newhall Earthquake

		BEST J_3 (DES1)		BEST J_5 (DES2)	
J	**Definition**	**ACT**	**SACT**	**ACT**	**SACT**
J_1	Peak Base Shear	0.984	1.169	0.704	1.138
J_2	Peak First Floor Shear	1.190	1.358	0.829	1.228
J_3	**Peak Base Drift**	**0.925**	**0.880**	1.153	0.783
J_4	Peak Interstory Drift	0.860	0.791	0.958	0.734
J_5	**Peak Absolute Floor Acceleration**	0.841	1.543	**0.548**	**1.444**
J_6	Peak Controller Force	0.490	0.527	0.479	0.522
J_7	RMS Base Drift	0.885	0.770	1.127	0.716
J_8	RMS Absolute Floor Acceleration	0.953	1.431	0.643	1.399
J_9	Energy Absorbed by the Con. Dev.	0.348	0.418	0.082	0.421
J_{10}	Normalized Peak Controller Force	0.087	0.094	0.085	0.093
J_{11}	RMS Floor Drifts	0.790	0.744	0.982	0.709

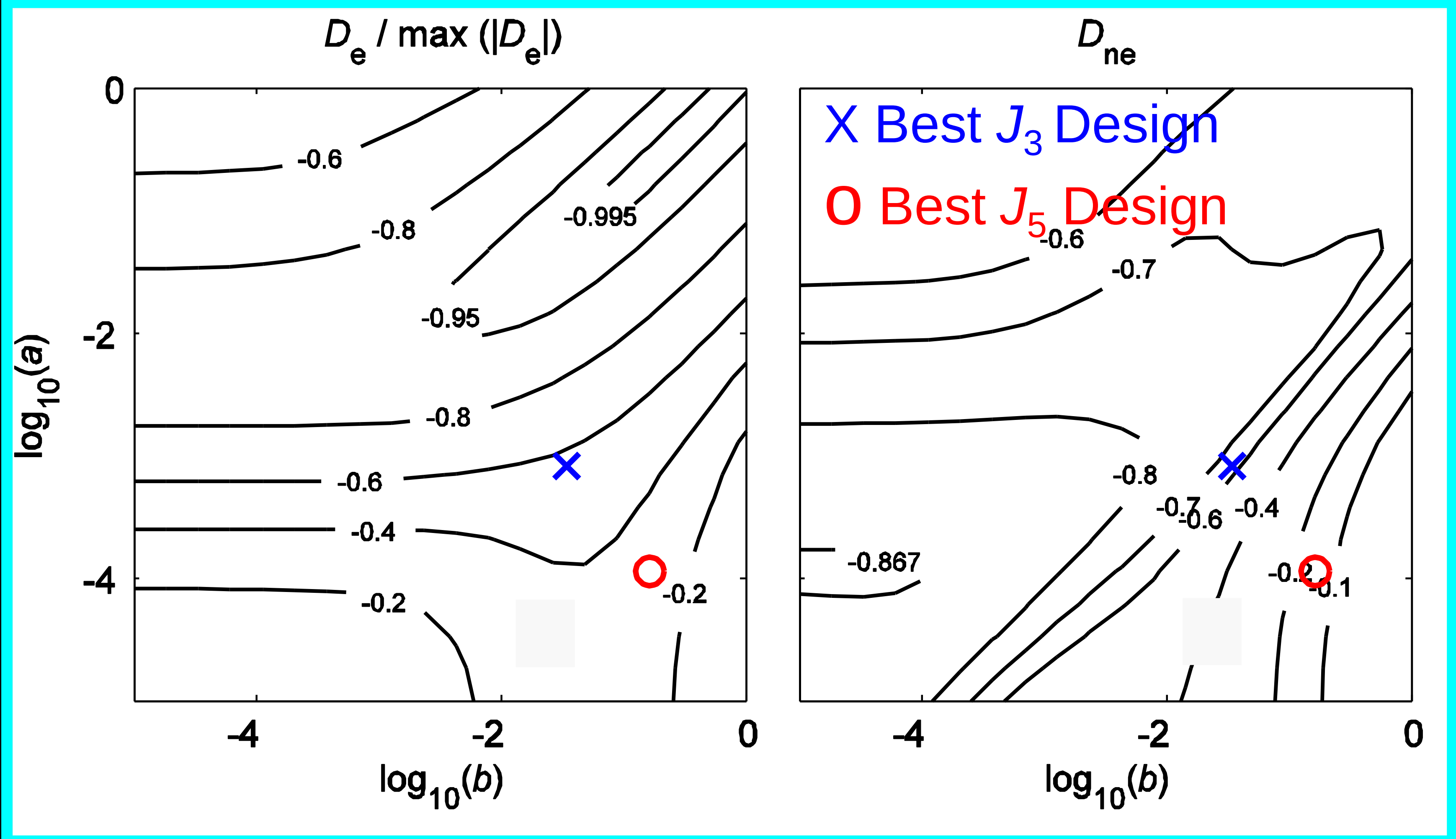
Nonlinear BM Structure: Active System

- Newhall Earthquake



Nonlinear BM Structure: Active System

- Stochastic Indices



Discussion: Major Observations

- Two classes of dissipativity indices are identified:

	Characteristics of the Control Force	D
GROUP 1	Normalized mean energy dissipation rate	D_{ne}
	Percentage of dissipative forces	$D_{\%}$
	Probability of dissipative forces	D_{p}
GROUP 2	Energy dissipation rate	D_{e}
	Energy dissipated by the device	J_9

Discussion: Major Observations

- Proposed dissipativity indices effectively identify the dissipativity nature of a controller and has many benefits over other indices
  - They have more general form
  - Suitable for mathematical investigation (e.g. LMI-LQR)
  - A great benefit for a realistic structural control problem is time-efficiency; this is a major issue in semiactive design

System	Time
Proposed Indices (Active Lyapunov)	< 1 sec
Active Earthquake (linear and nonlinear)	> 10 mins
Semiactive Earthquake (linear and nonlinear)	> 4 hours

Discussion: Major Observations

- Performance of the semiactive system is dependent to both the primary controller performance (a corresponding fully active system) and dissipativity of the primary controller.
  - A highly dissipative control design does not necessarily result a high semiactive performance; for these control designs, the primary controller performance may be low.

Conclusions

- Dissipativity indices are introduced to quantify the dissipativity nature of a primary control force in clipped optimal control of smart dampers.
- Proposed indices are utilized to modify an LQR problem to achieve control forces various dissipativity levels
- Dissipativity-performance relations of simple and complex structural systems are investigated with the dissipativity indices are proposed modified LQR theory

Conclusions: It is shown that…

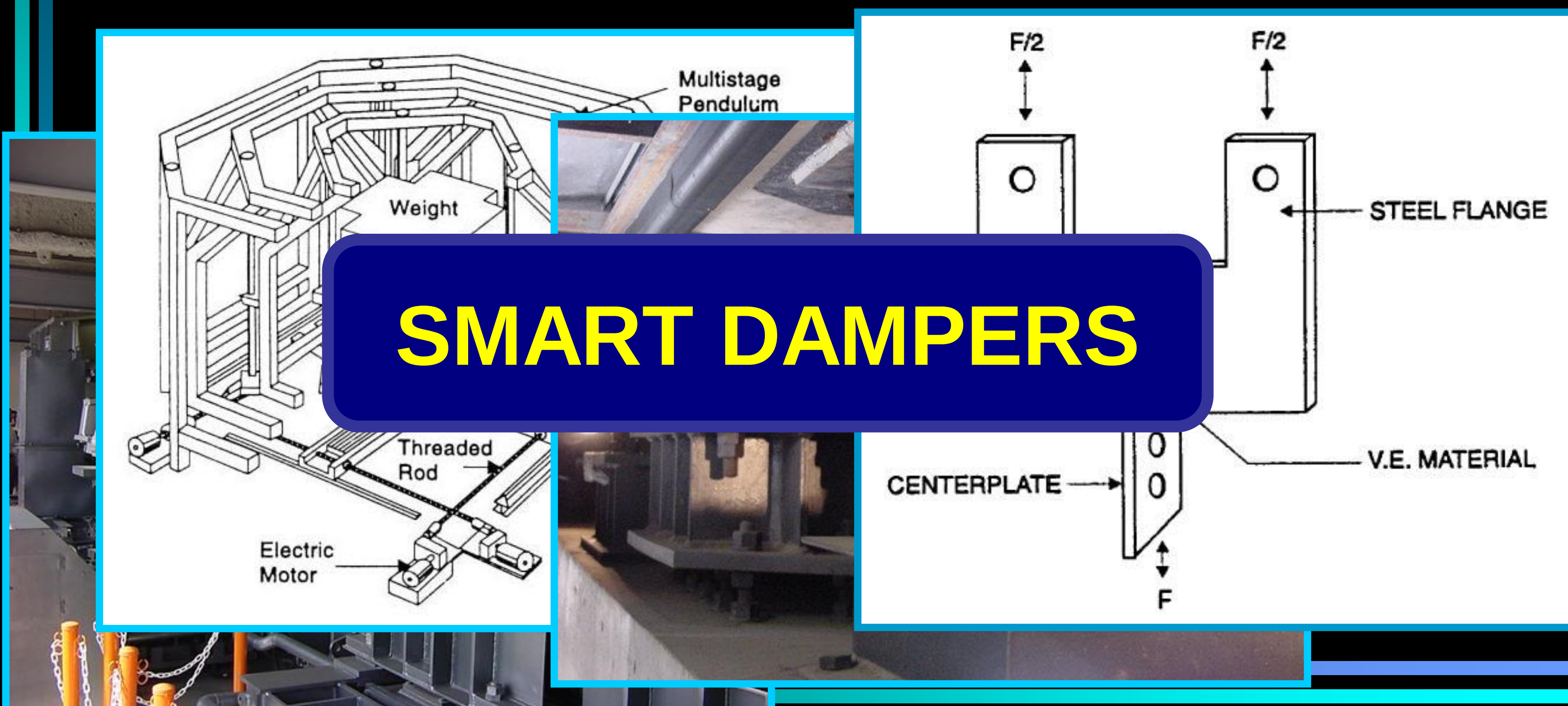
- Dissipativity indices can effectively identify the dissipativity nature of the primary controller, which helps to identify control designs with higher dissipativities and, therefore, designs that are more suitable for smart dampers
- Proposed modified LQR can be used in low-order systems to modify the dissipativity levels of the control force for better semiactive performance, yet it is not recommended to use for complex systems as the available LMI-solver is inefficient
- Dissipativity analysis provides a generalized, time-efficient tool for a semiactive design, which is very time consuming in complex structural systems.

Thanks

- Thank you for your attention

Introduction: Structural Control

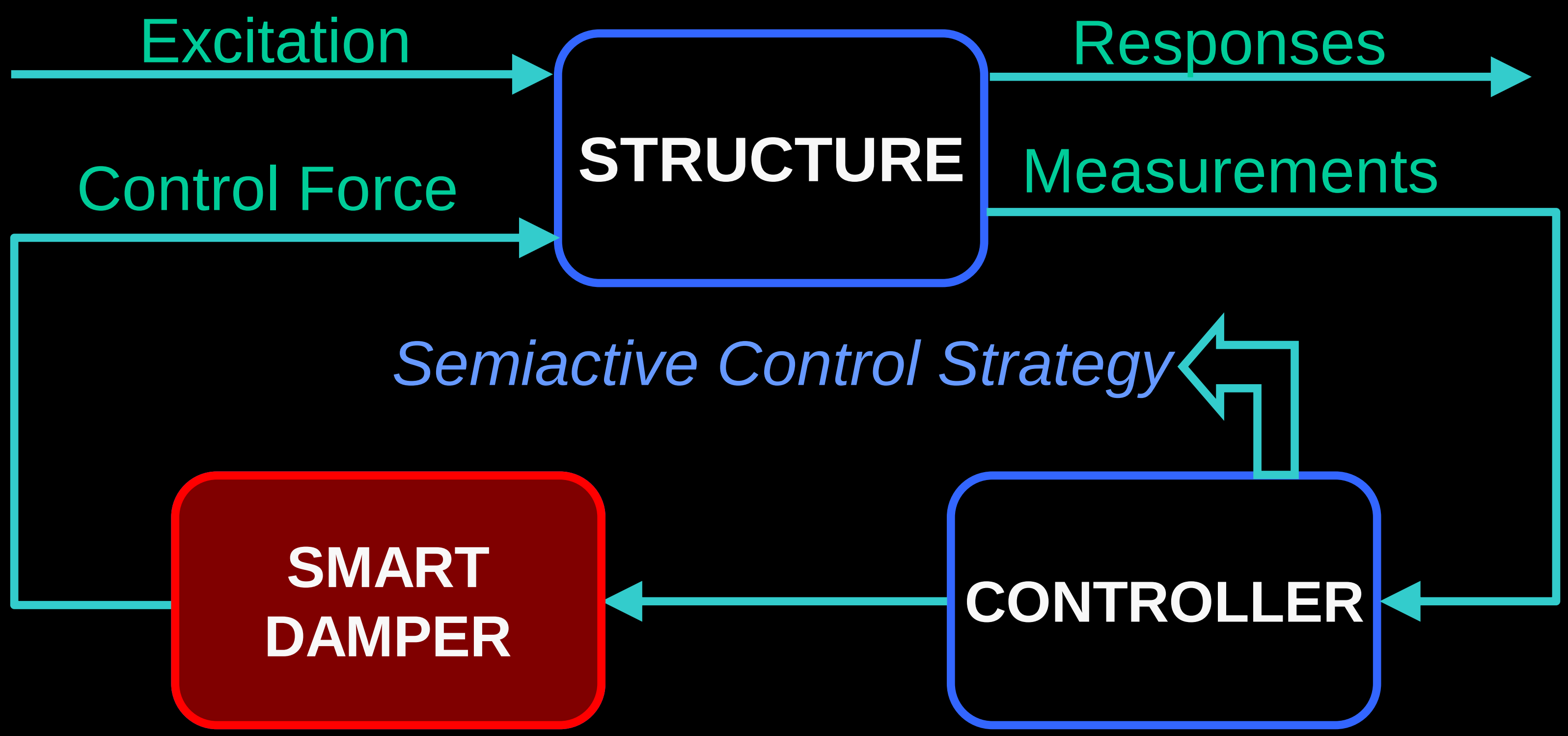
Active Control	Passive Control	Semiactive Control
⇩	⇩	⇩
Wind induced vibration mitigation of high rise buildings	Seismic isolation of buildings and bridges, energy dissipation in structures	Seismic and wind induced vibration of structures



SMART DAMPERS

Introduction: Control of Smart Dampers

- SemiActive Control



Smart Damper : Can only exert dissipative forces

Active Device : Theoretically, can apply *any* type force

Introduction: Organization

- Introduce dissipativity indices to quantify dissipativity
- Propose a method to modify a control theory to achieve controllers with various dissipativity levels
  - Use linear matrix inequality (LMI) methods to represent a linear quadratic regulator (LQR) in terms of matrix inequality constraints
  - Represent the dissipativity indices in terms of matrix inequalities and append to LMI-LQR problem
- Investigate structural systems common in civil engineering
  - 2-DOF models:
    - A shear building
    - An elevated highway bridge
    - Utilize the proposed LMI-LQR controller to modify the dissipativity of the primary controller in clipped optimal control
  - A realistic structural control problem: base isolated benchmark building
    - Linear isolation
    - Nonlinear isolation
    - Extensive dissipativity performance analysis

Dissipativity Indices

- Strictly dissipative force
  - Consider a force $f(t)$ applied to a point x_0 on the structure. $f(t)$ is called *strictly dissipative force* if the rate of energy flow is negative for all t ≥ 0.

$$\boxed{\begin{aligned} f(t)v(t) \le \varepsilon(t) < 0, \quad \text{for all} \quad t \le 0 \Leftrightarrow \\ f(t) \ \text{ is strictly dissipative} \end{aligned}}$$

$v(t)$ is the velocity of point x_0

$\varepsilon(t)$ is a strictly negative function

  - The force and the velocity have opposite directions
  - A damper force is a strictly dissipative force

LMI Representation of an LQR Problem

- Linear Quadratic Regulator (LQR) Problem
  - Consider a linear time-invariant system

$$\dot{\mathbf{q}} = \mathbf{A}\mathbf{q} + \mathbf{B}\mathbf{u} + \mathbf{E}\mathbf{w}$$

$$\mathbf{z} = \mathbf{C}_z\mathbf{q} + \mathbf{D}_z\mathbf{u} + \mathbf{F}_z\mathbf{w}$$

Find the constant control gain **K** such that

$$\min_{\mathbf{K}} \; E[\mathbf{z}^T\tilde{\mathbf{Q}}\mathbf{z} + \mathbf{u}^T\tilde{\mathbf{R}}\mathbf{u} + \mathbf{z}^T\tilde{\mathbf{N}}\mathbf{u} + \mathbf{u}^T\tilde{\mathbf{N}}^T\mathbf{z}]$$

$$\text{subject to } \; \text{[State Eq'n]} \; \text{ and } \mathbf{u} = -\mathbf{K}\mathbf{q}$$

$\tilde{\mathbf{Q}} \geq 0 \quad \tilde{\mathbf{R}} \geq 0$ are symmetric weighting matrices

LMI Representation of an LQR Problem

- Another form of LQR Problem

$$\min_{\mathbf{K}} \; E[\mathbf{q}^T\mathbf{Q}\mathbf{q} + \mathbf{u}^T\mathbf{R}\mathbf{u} + \mathbf{q}^T\mathbf{N}\mathbf{u} + \mathbf{u}^T\mathbf{N}^T\mathbf{q}]$$

$$\text{subject to} \quad \dot{\mathbf{q}} = \mathbf{A}\mathbf{q} + \mathbf{B}\mathbf{u} + \mathbf{E}\mathbf{w} \quad \text{and } \mathbf{u} = -\mathbf{K}\mathbf{q}$$

where

$$\mathbf{Q} = \mathbf{C}_z^T\tilde{\mathbf{Q}}\mathbf{C}_z \qquad \mathbf{N} = \mathbf{C}_z^T\tilde{\mathbf{Q}}\mathbf{D}_z + \mathbf{C}_z^T\tilde{\mathbf{N}}$$

$$\mathbf{R} = \tilde{\mathbf{R}} + \mathbf{D}_z^T\tilde{\mathbf{Q}}\mathbf{D}_z + \mathbf{D}_z^T\tilde{\mathbf{N}} + \tilde{\mathbf{N}}^T\mathbf{D}_z$$

To be well-posed, LQR weights must satisfy

$$\mathbf{W} = \begin{bmatrix} \mathbf{Q} & \mathbf{N} \\ \mathbf{N}^T & \mathbf{R} \end{bmatrix} \geq 0 \quad \text{and} \quad \mathbf{R} > 0$$

Dissipativity Constraints

D_{p}-based constraint:

$$\frac{\cos^{-1}\left(\dfrac{-\mathbf{FPC}_{\mathrm{v}}^{\mathrm{T}}}{\sqrt{\mathbf{FPF}^{\mathrm{T}}}\sqrt{\mathbf{C}_{\mathrm{v}}\mathbf{PC}_{\mathrm{v}}^{\mathrm{T}}}}\right)}{\pi} < \gamma_{\mathrm{p}} \quad \text{where} \quad 0 \le \gamma_{\mathrm{p}} \le 1$$

P : State covariance matrix

D_{e}-based constraint:

$$-\mathbf{FPC}_{\mathrm{v}}^{\mathrm{T}} < \gamma_{\mathrm{e}}$$

D_{ne}-based constraint:

$$\frac{-\mathbf{FPC}_{\mathrm{v}}^{\mathrm{T}}}{\sqrt{\mathbf{FPF}^{\mathrm{T}}}\sqrt{\mathbf{C}_{\mathrm{v}}\mathbf{PC}_{\mathrm{v}}^{\mathrm{T}}}} < \gamma_{\mathrm{ne}} \quad \text{where} \quad -1 \le \gamma_{\mathrm{ne}} \le 1$$

Dissipativity Constraints

D_e-based constraint:

$$\boxed{-\mathbf{FPC}_v^T < \gamma_e}$$

$$-\mathbf{FSC}_v^T < \gamma_e \quad \text{where} \quad (\mathbf{A}-\mathbf{BF})\mathbf{S}+\mathbf{S}(\mathbf{A}-\mathbf{BF})^T+\mathbf{EE}^T=0$$

$$\boxed{-\mathbf{FSC}_v^T < \gamma_e^L}$$

- This constraint is weaker than the "strictly dissipative force" condition ($u_a \; v_d < 0$)
  - Mean value is used
  - Lyapunov matrix (**S**) is used instead of the covariance matrix (**P**)

Dissipativity Constraints

D_{ne}-based constraint:

$$\boxed{\frac{-\mathbf{FPC}_{\text{v}}^{\text{T}}}{\sqrt{\mathbf{FPF}^{\text{T}}}\sqrt{\mathbf{C}_{\text{v}}\mathbf{PC}_{\text{v}}^{\text{T}}}} < \gamma_{\text{ne}} \quad \text{where} \quad -1 \le \gamma_{\text{ne}} \le 1}$$

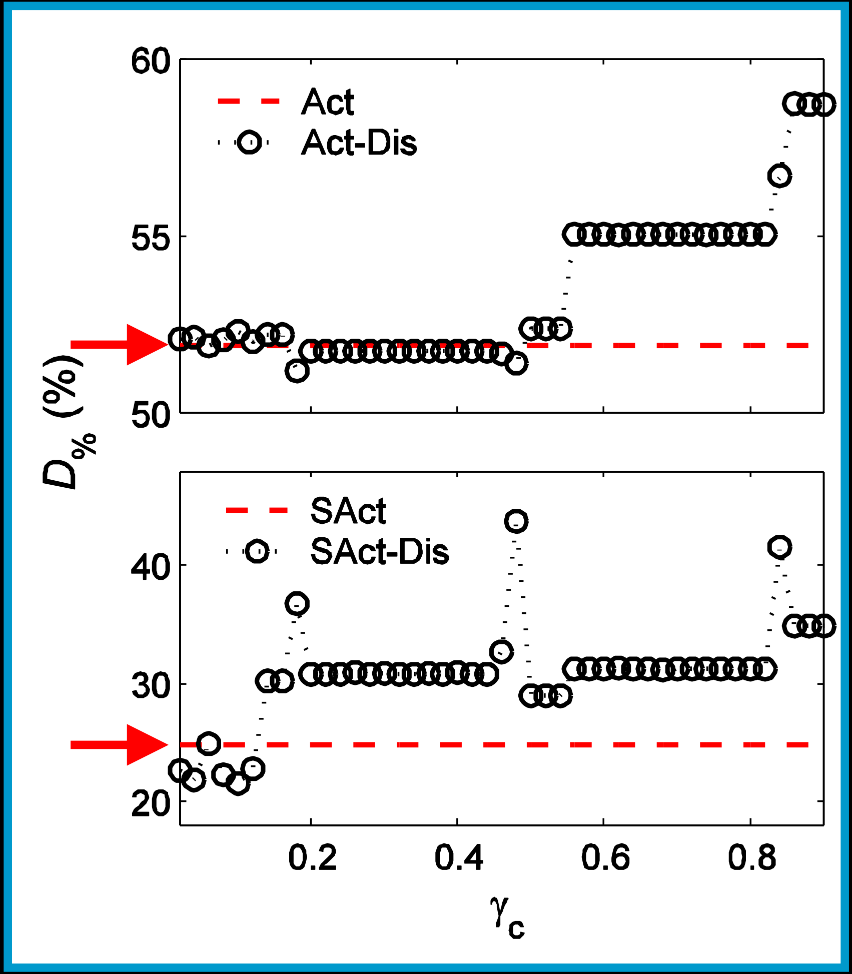
$$\frac{-\mathbf{FSC}_{\text{v}}^{\text{T}}}{\sqrt{\mathbf{FSF}^{\text{T}}}\sqrt{\mathbf{C}_{\text{v}}\mathbf{SC}_{\text{v}}^{\text{T}}}} < \gamma_{\text{ne}}$$

where $(\mathbf{A}-\mathbf{BF})\mathbf{S}+\mathbf{S}(\mathbf{A}-\mathbf{BF})^{\text{T}}+\mathbf{EE}^{T}=0$

$$\boxed{\frac{-\mathbf{FSC}_{\text{v}}^{\text{T}}}{\sqrt{\mathbf{FSF}^{\text{T}}}\sqrt{\mathbf{C}_{\text{v}}\mathbf{SC}_{\text{v}}^{\text{T}}}} < \gamma_{\text{ne}}}$$

2-DOF Systems: Highway Bridge

Normalized indices



- Originally, dissipativity levels of the active system is low
- Dissipativity levels in the semiactive system is lowered further from 52% to 25%.
- This reduces the performance further

Dissipativity Constraint Based on $D_{\mathbf{ne}}$

Consider the LMI-LQR problem

$$\min_{(\mathbf{Y},\mathbf{S},\mathbf{X})} \quad Tr(\mathbf{Q}^{1/2}\mathbf{S}\mathbf{Q}^{1/2}) + Tr(\mathbf{X}) - Tr(\mathbf{YN}) - Tr(\mathbf{N}^{\mathrm{T}}\mathbf{Y}^{\mathrm{T}})$$

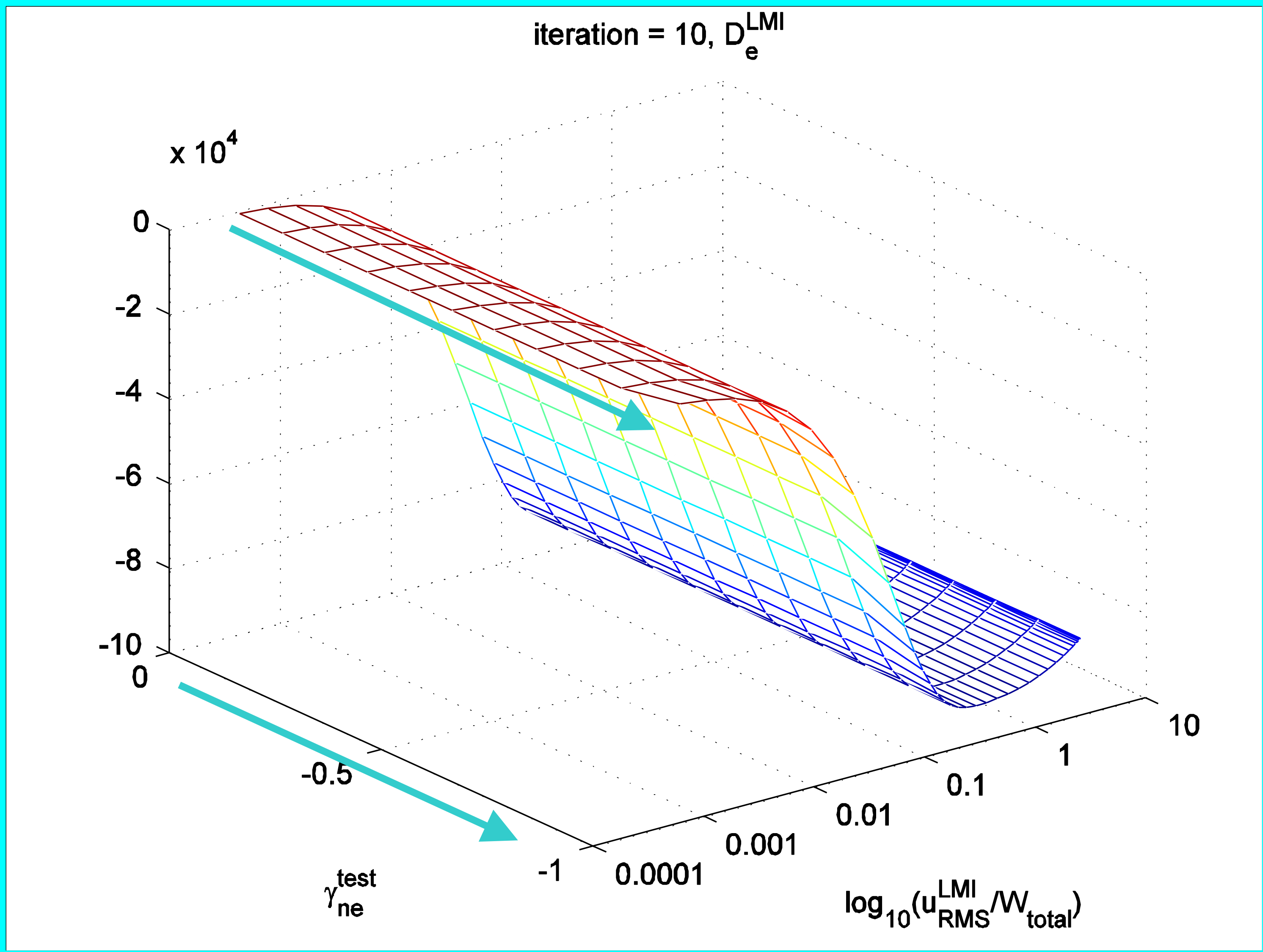
$$\text{subject to} \quad \mathbf{AS} - \mathbf{BY} + \mathbf{SA}^{\mathrm{T}} - \mathbf{Y}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{EE}^{T} < 0, \quad \begin{bmatrix} \mathbf{X} & \mathbf{R}^{1/2}\mathbf{Y} \\ \mathbf{Y}^{\mathrm{T}}\mathbf{R}^{1/2} & \mathbf{S} \end{bmatrix} > 0$$

$$\text{and} \quad -\mathbf{FSC}_{\mathrm{v}}^{\mathrm{T}} < \gamma_{\mathrm{e}}^{\mathrm{L}}$$

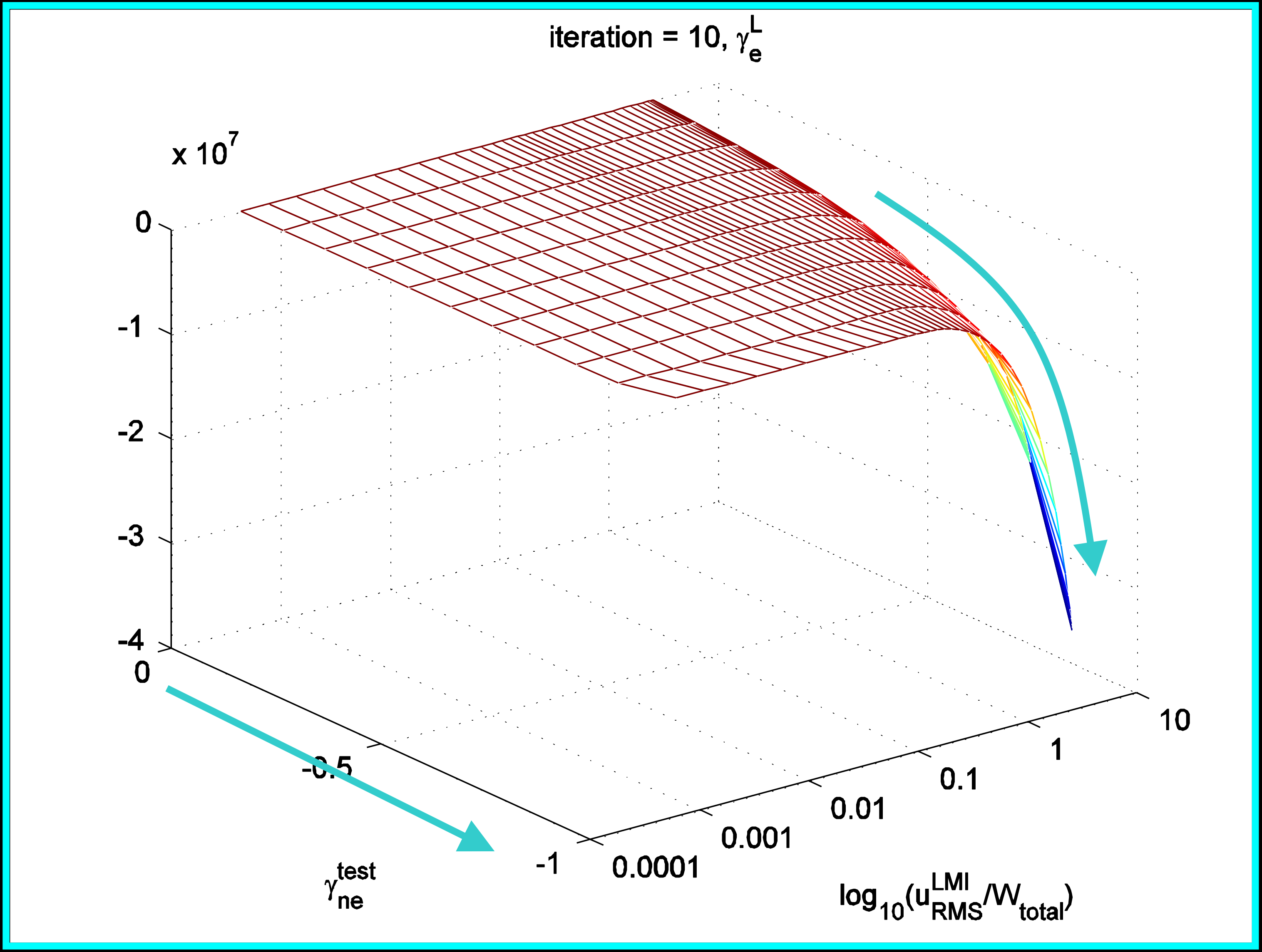
$$\frac{-\mathbf{FSC}_{\mathrm{v}}^{\mathrm{T}}}{\sqrt{\mathbf{FSF}^{\mathrm{T}}}\sqrt{\mathbf{C}_{\mathrm{v}}\mathbf{SC}_{\mathrm{v}}^{\mathrm{T}}}} < \gamma_{\mathrm{ne}}$$

- Nonlinear constraint, may be converted to BMI
- Solution set is not guaranteed to be convex;
  - No guaranteed algorithm exists for the numerical solution

2-DOF Systems: Shear Building (D_{ne})



2-DOF Systems: Shear Building (D_{ne})



BM Structure: Modelling

- Superstructure

$$\ddot{\boldsymbol{\eta}}_s^b + \tilde{\mathbf{C}}_s\dot{\boldsymbol{\eta}}_s^b + \tilde{\mathbf{K}}_s\boldsymbol{\eta}_s^b = -\boldsymbol{\Phi}_s^T\mathbf{M}_s\mathbf{R}_s\ddot{\mathbf{x}}_b^{abs}$$

- Rigid base and the isolation layer

$$\mathbf{M}_b\ddot{\mathbf{x}}_b + \mathbf{C}_b\dot{\mathbf{x}} + \mathbf{K}_b\mathbf{x}_b = -\mathbf{M}_b\mathbf{R}_b\ddot{\mathbf{x}}_g^{abs} + \mathbf{S}_c'\mathbf{u} + \mathbf{S}_f'\mathbf{f} + \mathbf{F}_s$$

- Base shear

$$\mathbf{F}_s = \mathbf{R}_s^T\mathbf{M}_s\left(\ddot{\mathbf{x}}_s^b + \mathbf{R}_s^T\ddot{\mathbf{x}}_b^{abs}\right)$$

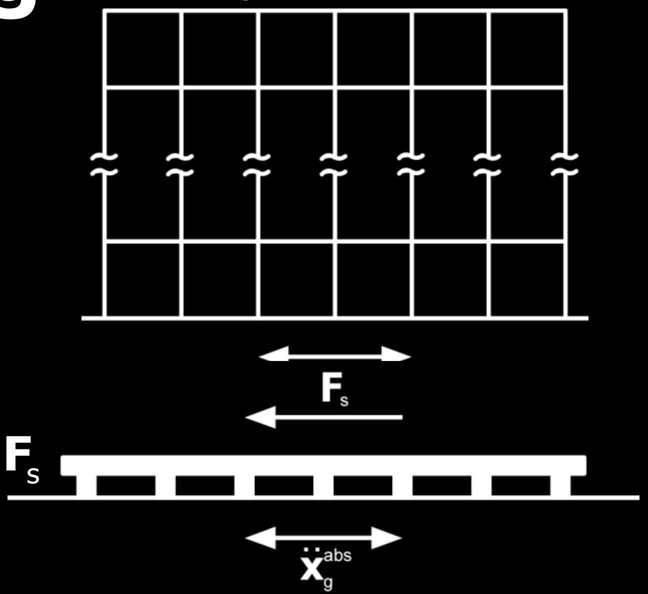
- Equation of motion

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{R}\ddot{\mathbf{x}}_g + \mathbf{S}_c\mathbf{u} + \mathbf{S}_f\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) \qquad \mathbf{x} = \begin{Bmatrix} \boldsymbol{\eta}_s^b \\ \mathbf{x}_b \end{Bmatrix}$$

- State-space form

$$\dot{\mathbf{q}} = \mathbf{A}\mathbf{q} + \mathbf{B}\mathbf{u} + \mathbf{E}\ddot{\mathbf{x}}_g + \mathbf{F}\mathbf{f}(\mathbf{q})$$

Superstructure



Rigid base and the isolation layer

$\boldsymbol{\eta}_s^b$: modal response of the superstructure wrt the base

$\mathbf{x}_b$: base displacement wrt the ground

Nonlinear BM Structure

- Active control design: Linearization techniques
  - Two goals:
    1. Obtain an equivalent linearized model of the nonlinear structure
    2. Design a linear controller for the equivalent linear model
  - Equivalent in which sense?
    - Equivalent model should represent response characteristics of the controlled nonlinear structure!
    - Need to know response of the controlled nonlinear structure
    - Need to know controller
    - Need to know equivalent linear model

➔ Design of *an equivalent linear model* and design of *a linear controller* are coupled problems

Nonlinear BM Structure

- Structural properties
  - Originally 31 rubber bearings and 61 friction bearings
  - Lead rubber bearings are used instead of frictional bearings

→ 31 rubber bearings and 61 lead rubber bearings

31 rubber (linear) 61 lead (bilinear) rubber (linear)	→	91 rubber (linear) 61 lead plugs bilinear

91 linear elements (rubber) :

Stiffness: 919.4 kN/m

Damping: 101.4 kN.s/m

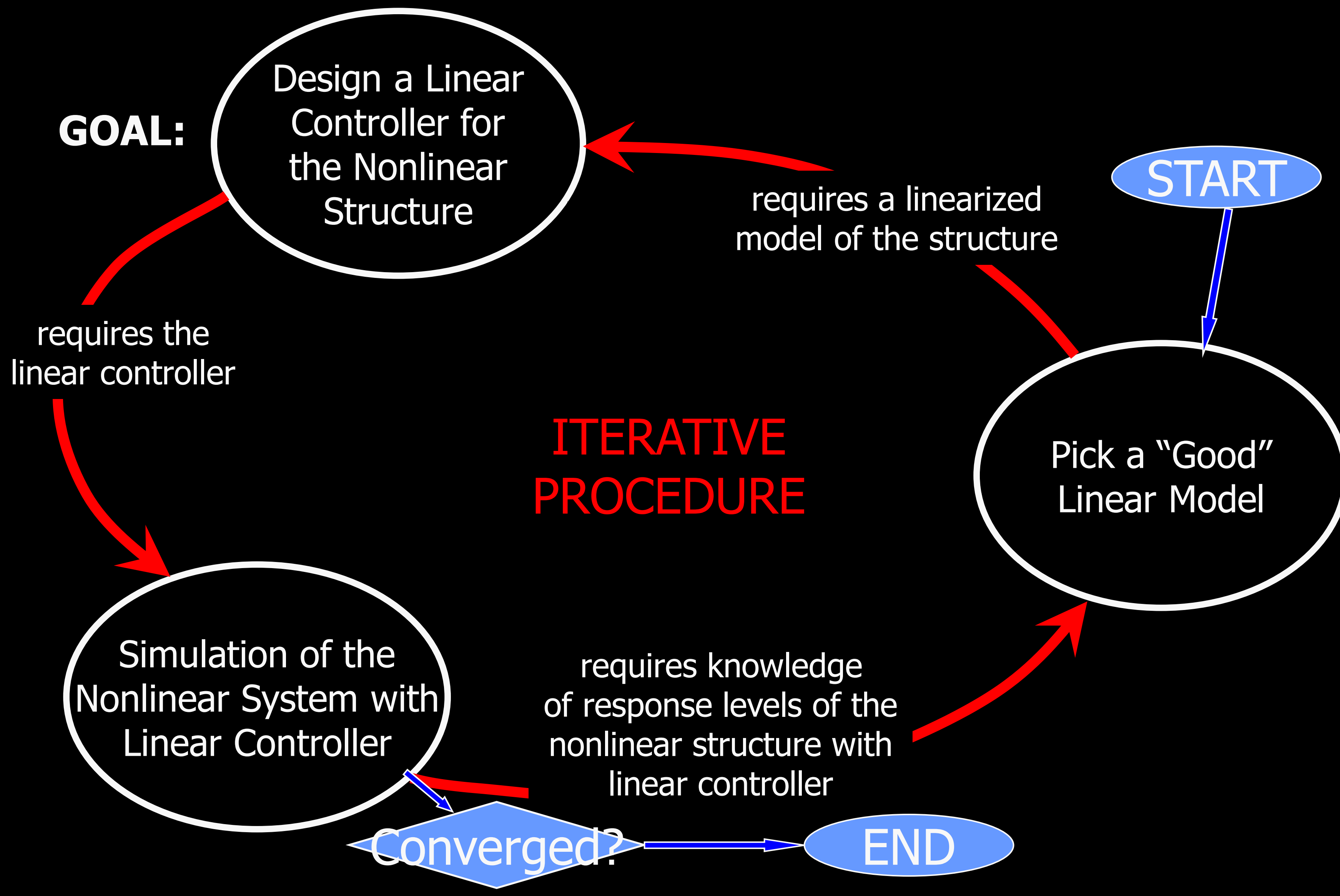
61 bilinear elements (lead plugs) :

Preyield stiffness: 5546.7 kN/m, Postyield stiffness: 0 kN/m

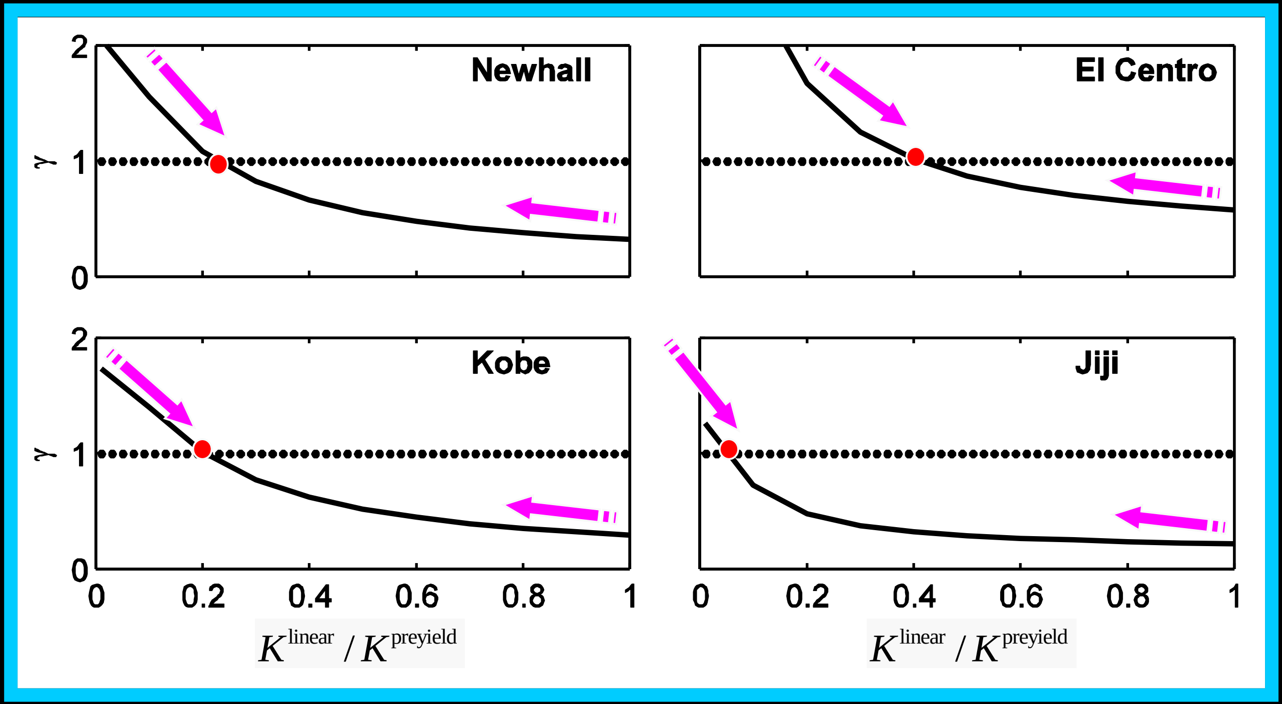
Yielding force: 132.5 kN

Damping: 207 kN.s/m (!)

Nonlinear BM Structure: Control Design



Nonlinear BM Structure: Gamma Iteration



Variation of γ_f for several values of

$$\frac{K^{linear}}{K^{preyield}}$$

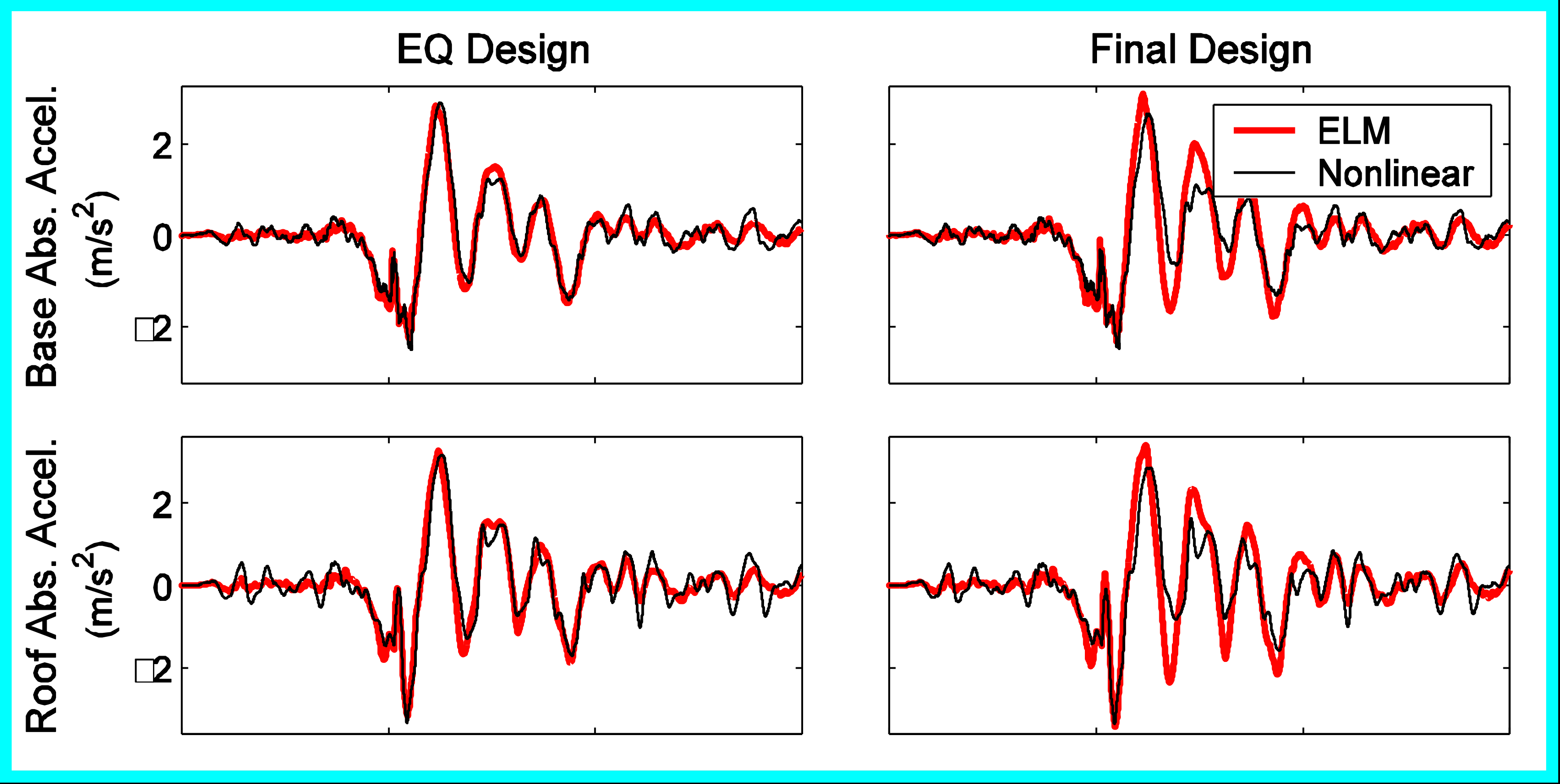
Convergence guaranteed

$$\boxed{K_{final}/K_{preyield}=0.175}$$

(Average of seven earthquake iterations)

Nonlinear BM Structure: ELM vs Nonlinear

- Jiji Earthquake



Nonlinear BM Structure: ELM vs Nonlinear

- Jiji Earthquake

Base Disp.

Nonlin Force

Hysteretic Behavior of the Lead Plugs Only